OPRE 504: Research in Mathematical Finance I
Tentative Syllabus, Fall 2001

Instructor: Peter Ritchken

Text : "Options, Futures and other Derivative Securities'' by J. Hull, Prentice Hall

- This book is very readable and provides a working knowledge of how derivative securities can be analyzed. I will assign chapters to read from this book and I also will assign some homework problems from the book.

- The book is geared towards advanced MBA students and does not provide as much rigor as is required from this class. However, the book does provide an excellent breadth of material, and will allow us to cover the basics in short order.

- You may also want to read my notes for BAFI 430 which you can download from my web page. http://weatherhead.cwru.edu/faculty_research/homes/ritchken**.**

- In class I will provide notes that are more technical.

Recommended Readings: There are some excellent books that may be useful to you. These include:

- "Dynamic Asset Pricing Theory" by D. Duffie. Princeton University Press, 1992
- "Mathematics of Financial Derivatives: A student Introduction" by Paul Wilmot, et al. Cambridge University Press, 1995
- "Mathematical Models of Financial Derivatives", Y. Kwok, Springer.
- "Introduction to Mathematical Finance", S. Pliska, Blackwell.
- "Investment Under Uncertainty", A. Dixit, S. Pindyck, Princeton
- "Financial Calculus", Baxter, M. and A. Rennie, Cambridge University Press,1996
- Mathematics of Financial Derivatives, S. Neftci, Academic Press, 1996
- There are lots of good readings on web. sites. I will provide these addresses.

Grading:			
	Homework	20%	(approx. one every 2 weeks)
	Project or Paper	20%	
	Final Exam	60%	

The final exam is on Wed. October 17th.

Objective

The field of financial economics has mushroomed both in scope and applications in recent years and these developments have a profound effect on many methodologies in Operations Research and Operations Management. This course attempts to capture some key elements of this theory , with a strong emphasis on applications in valuation, and risk management.

The focus of this first course will be on no arbitrage pricing methodologies rather than on equilibrium pricing. The latter deals with how individual buyers and sellers arrive at prices in a market. Equilibrium is reached when these agents cannot improve their positions by additional trading. The theory depends on using utility to describe the welfare of agents. No arbitrage pricing theory shows how prices of some claims are obtained when the price dynamics of related primitive securities are already known. The prices are obtained as expected present values by using a special set of probabilities known as risk neutral or equivalent martingale probability measure. This methodology can be traced back to the Black and Scholes option pricing model, and will be very carefully examined.

In this 1.5 credit hour class, our focus will be on the development of the basic economic and mathematical tools used in financial economics. After taking this class you will be able to read the technical papers on valuation. Moreover, you will be able to apply these ideas to an array of interesting problems

The mathematical concepts that are used in this class are fairly deep. The math department teaches several very good classes that fully develop these tools. I will provide heuristic developments of the theory, but our orientation is clearly geared towards using these tools in financial applications.

Project

Every student will be expected to read and analyze a published research paper. After finding a paper you must have it approved by me. You will be responsible for putting together a set of overheads that articulate the important points of the paper. Where appropriate, you may program the model, and provide illustrations of the main ideas. Of course, using related material, and building around the central ideas of the paper will be rewarded. Time permitting, you may present the material to the class.

The grade for the project will be determined in large part by your ability to convey the important messages of the paper.

Very Tentative Class Schedule

Below is a rather ambitious outline that covers the basics of mathematical finance. In a 1.5 credit hour class, I am not sure how much we will cover. We will see how it goes and adjust the schedule accordingly!!!!

Module 1: Asset pricing Theory with Consumption Based Models

- One Period Model and the First Order Conditions.
- Marginal rate of substitution and the stochastic discount factor.
- Contingent Claims Markets
- Classical issues in finance.
    - The economics of interest rates
    - Risk adjustments
    - Systematic vs. Idiosyncratic risk
    - Expected return- beta representations
    - Present Value Relationships
- Contingent Claims Markets
    - Contingent Claims
    - Risk neutral Probabilities
    - Investors First Order Conditions –Revisited
    - Risk Sharing

Module 2 : No Arbitrage Pricing Theory:

Single Period Models

- Arbitrage and the Fundamental Theorem of Asset pricing
- Risk Neutral probability measures
- Valuation of Cash Flows
- Market Completeness
- Applications

Multiperiod Models

- Self Financing Trading Strategies
- The Fundamental Theorem of Asset pricing
- Applications

Module 3: Futures and Options (see Hull, and Ritchken)

Futures and Forward Contracts

- The Cost of Carry Model
- Pricing Futures and Forwards
- Relationship between futures and forwards

Options

- Definitions
- Arbitrage Relationships (See Merton Paper)

Module 4: Pricing Options Using Binomial lattices (see Hull, and Ritchken)

- The single period model
- The multi-period model
- Self financing trading strategies
- The Black Scholes equation
- Extensions and Applications. (See Cox, Ross, Rubinstein Paper)

Module 5: The Binomial Martingale Representation Theorem (see my notes)

- Binomial processes and measures
- Filtrations and claims.
- Martingales
- Conditional expectations and Martingales.
- The Binomial representation theorem.
- Discounted stock prices and discounted claims.
- Replicating process and the Binomial representation theorem.
- Summary, the importance of martingales.

Module 6: Martingales in Continuous Time and Stochastic Integrals (see my notes)

- First Order Variation, Quadratic Variation and pth order variation of Functions.
- Quadratic variation of differentiable functions
- Quadratic variation of continuous Martingale trajectories
- Quadratic variation of Brownian motion
- Examples of Martingales
- Our first stochastic integral
- Ordinary Riemann and Riemann Stieltjies Integrals
- The Ito Stochastic Integral for Simple Processes
- Properties of the Ito Stochastic Integral
- Ito Integral for General Integrands.

Module 7: Ito's Lemma and Stochastic Calculus (see my notes)

- Newtonian differentials and ordinary differential equations
- Stochastic differentials and stochastic differential equations
- Solving SDEs.
- Ito's lemma and examples
- Rules for differentiation
- Generalized Ito's lemma
- Examples
- Computing expectations and Variances
- Solving Ito-stochastic differential equations using Ito's lemma
- The Ornstein Uhlenbeck process
- General linear Differential Equations
- Connection between SDEs and PDEs
- Feynman Kac Theorem.
- Numerical Solutions of SDEs

Module 8: Change of Measure and the Martingale Representation Theorem (see my notes)

- The Radon Nikodym Derivative
- Expectation and the Radon Nikodym Process
- Girsanov's Theorem 1
- Girsanov's Theorem II and equivalent martingale measures
- The Martingale Reprenentation Theorem
- Black Scholes Revisited.

Here is a list of some of the classic papers that we may read:

Merton (1973), *Theory of Rational Option Pricing*, Bell Journal of Economics and management Science, 1973, 141-183

Black and Scholes (1973), *The pricing of Options and Corporate Liabilities*, Journal of Political Economy, 637-659.

Cox, Ross, Rubinstein (1979), *Option Pricing: A Simplified Approach*, Journal of Financial Economics, 229-263

Harrison and Pliska (1981), *Martingales and Stochastic Integrals in the Theory of Continuous Trading*, Stochastic processes and their Applications, 215-260

Brennan and Schwartz (1978), *Finite Difference Methods and Jump processes arising in the Pricing of Contingent Claims: A Synthesis*, Journal of Financial and Quantitative analysis, 462-474.

Geske(1979), *The Valuation of Compound Options*, Journal of Financial Economics, 63-81

Heath, Jarrow and Morton (1992), Bond Pricing and the Term Structure of Interest rates: A new methodology. Econometrica, 77-105.