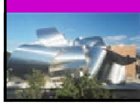
Pricing Options on
Dividend paying stocks, FOREX, Futures, Consumption Commodities



- The Black-Scoles Model
- The Binomial Model and Pricing American Options
- Pricing European Options on dividend paying stocks
- Pricing European Options on Stock Indices
- Pricing European Options on FOREX
- Pricing European Options on Futures
- Pricing European Options on consumption commodities
- Pricing American Options on the above.
- Pricing simple real options

European Call Options on a Stock

The Black Scholes Equation

$$C_0 = H^* S_0 - B^*$$

where

$$H^* = N(d_1)$$
$$B^* = Xe^{-rT}N(d_2)$$

and

$$d_1 = [ln(S_0/X) + (r + \sigma^2/2)T]/\sigma\sqrt{T}$$
$$d_2 = d_1 - \sigma\sqrt{T}.$$

$N(x)$ is the probability that a standard normal random variable (usually represented by Z is less than x.

Example: Pricing a European Call Option Using The Black Scholes Model

- Consider the theoretical price of a 3 month call option on a stock priced at \$50. The strike price is 45, the riskless rate is 6% and the volatility is 20% per year. Hence, $S(0) = 50, X = 45, T = 0.25, \sigma = 0.20$, and $r = 6\%$ Then, the Black Scholes call price can be computed as follows.

$$d_1 = [\ln(S_0/X) + (r + \sigma^2/2)T]/\sigma\sqrt{T} = 0.65$$
$$d_2 = d_1 - \sigma\sqrt{T} = 0.426$$

Then, using standard normal tables, $N(d_1) = 0.742, N(d_2) = 0.665$, and we obtain

$$H^* = N(d_1) = 0.742$$
$$B^* = Xe^{-rT}N(d_2) = 45e^{(-0.06)(0.25)}(0.665) = 29.48$$
$$C(0) = H^*S(0) - B^* = 50(0.742) - 29.48 = \$7.62$$

What do you need to price an option

- Underlying S(0)
- Strike, X
- Time to expiration, T
- Riskless rate, r
- Volatility

- Estimating volatility
  - Use Historical data
  - Use Implied Volatility

Option Models

- Give us Theoretical Prices, when none exist.
- Give us prices which we can use to compare with market prices.
- Allows us to establish strategies. (Buy underpriced/sell overpriced)
- Gives us the replicating portfolio.
- Allows us to imply out volatility information.
- Test of models and market efficiency.
- Models can be used to price corporate securities and many real assets

Historical Volatility vs Implied Volatility

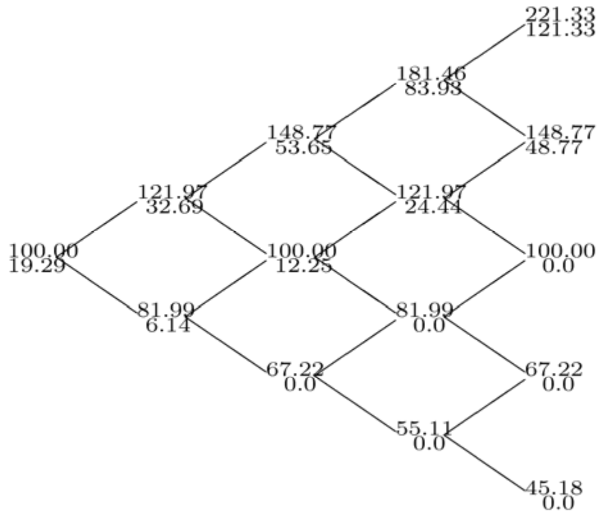
- What is implied volatility?
- Is it a good predictor of actual future volatility?
- Which Implied Volatility?
- Tests of Option Models
- The Volatility Smile.

Example : Pricing a Call Option Using a 4 - Period Binomial Approximation

Consider a 1 year at-the-money American call option on a non-dividend paying stock. The stock price is \$100, the risk free rate is 10%, and the annual volatility is 39.72% per year. Using four partitions, $n = 4, T = 1.0$, yields $\Delta t = 0.250$ and

$$\begin{aligned} u &= e^{\sigma\sqrt{\Delta t}} = 1.2197 \\ d &= e^{-\sigma\sqrt{\Delta t}} = 0.8199 \\ \theta &= \frac{e^{r\Delta t} - d}{u - d} = 0.51379 \end{aligned}$$

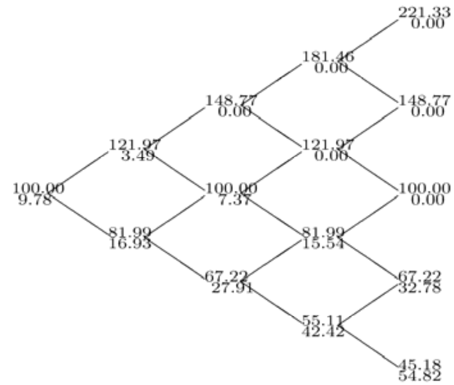
Figure 15: Stock and Call Option Prices



Example: Pricing a Put Option Using a 4-Period Binomial Approximation

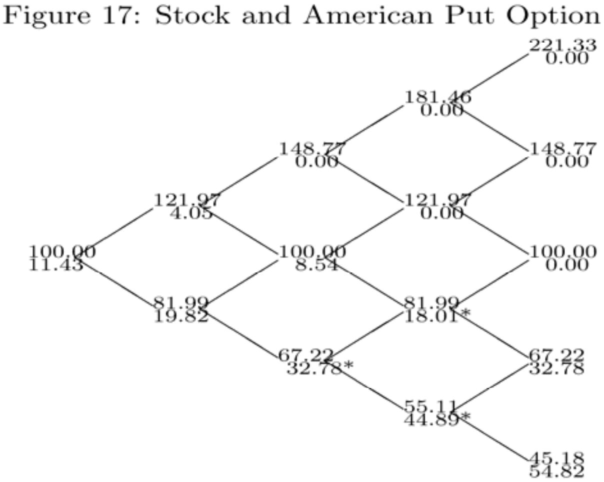
- Consider a 1 year at-the-money European put option on the previous stock. Following the backward recursion from the terminal period yields the put prices indicated below the stock price.
- Notice that the option price can fall below the intrinsic value, since early exercise is not permitted.
- Specifically, when the option is deep in-the- money, early exercise may be advantageous since the strike price can be obtained early and can be used to generate interest income.

Figure 16: Stock and Put Option Prices



Example: Pricing an American Put Option

Figure 17: Stock and American Put Option Prices



- To price an option in the Binomial lattice we replace the growth rate of the stock by the by the riskless rate.
- The lattice converges to a Geometric Wiener process.
- For European options we can use simulation, where we replace the growth rate by the riskfree rate.
- Simulation, the Binomial lattice and the Black Scholes model, give the same results for pricing an European option.
- The Binomial lattice is good for American options.
- Simulation is good for exotics.

Options on Stocks paying dividends

- Assume a stock pays out d at a time before the expiration date,
- S(0) = PV(d) + PV(all other future expected dividends)
- The call holder has a claim on the second component.
- S(0) = PV(d) + G(0)
- After the dividend at date t say, S(t) = G(t)
- The call holder has a claim on G(T). The current value of G(T) is G(0).
- Use Black Scholes with G(0) not S(0).

Options on Stocks paying Continuous dividends.

- Total expected return = dividend yield + price appreciation
- Need to reduce the growth rate by the dividend yield.
- So for a simulation, we have:

$$S(t) = S(0)e^{\alpha T + \sigma\sqrt{T} Z_T}$$

$$\alpha = r - q - \sigma^2 / 2$$

European options on stocks paying continuous dividends

- A stock valued at S(0) paying dividends continuously has the same value at date T as a stock paying no dividends would have if its date 0 value was G(0)= S(0)exp(-qT)
- To price an European option use G(0) instead of S(0)

Example

- The dividend yield on the S&P 500 over the next month is estimated as 4.2% per year. The index is at 300. The following data is given
- Strike 300
- R= 6%
- T = 1 month
- G(0) = 300 exp(-0.042/12) = 298.95
- C(0) = 10.47.

European Options on FOREX

$$\mu = r - q$$

$$\alpha = \mu - \sigma^2 / 2$$

$$q = r_F$$

$$G(0) = S(0)e^{-r_F T}$$

- Use G(0) instead of S(0) in the BS equation

European options on Futures (The Black Model)

- What should the expected price appreciation on a Futures contract be?
- Hint: The initial investment is zero?
- Yes! The growth rate should be zero!

$$\mu = r - q$$
$$\alpha = \mu - \sigma^2 / 2$$

So take q =r
G(0) = S(0) exp(-rT)

European Calls on Consumption Commodities

- A commodity pays a convenience yield
- The convenience yield is a non observable dividend
- Can be extracted from the forward price.
- Let u be the annualized convenience yield over the time period to the expiration date.
- Use G(0) = S(0)exp(-uT) in the Black Scholes Equation.

Pricing American Options on a lattice

- Compute u and d the same way.
- Use

$$p = \frac{e^{(r-q)\Delta t} - d}{(u-d)}$$

Watch out for early exercise.

Pricing a Real Option

- You have the option to buy a building for 1m dollars. The option expires in one year.
- The building provides a rental income of 5%
- The riskless rate is 8%
- What is the value of the option.

Pricing a Real Option

- You have the option to buy a building for 1m dollars. The option expires in one year.
- The building provides a rental income of 5%
- The riskless rate is 8%
- What is the value of the option.
- The current prurchase price is S(0) = 1m dollars.

- The growth rate of the building in a risk neutral setting is (8-5)%
- If the option is European, set G(0) = S(0)exp(-0.05).
- Use Black Scholes!

Conclusion

- How to Use Black Scholes
- Black Scholes and the Binomial lattice
- Dividends –discrete
- Dividends- continuous
- Pricing options on Indices, FOREX, Futures, Consumption commodities, real assets.
- American options and early exercise.
- Simulation, Lattices, and Black Scholes.