Interest Rate and Credit Risk Derivatives

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For my family

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Bond Price Arithmetic

The purpose of this chapter is:

- To review the basics of the time value of money. This involves reviewing discounting guaranteed future cash flows at annual, semiannual and continuously compounded rates.
- To learn how to handle cash flows that are unequally spaced, or where there are fractional periods of time to particular cash flows.
- To understand the market convention of quoting prices, computing accrued interest and communicating prices in a yield form.
- To set the stage for a deeper analysis of fixed income products.

1.1 FUTURE VALUE AND COMPOUNDING INTERVALS

Let $\$P$ be invested at a simple interest rate of $y\%$ per year for one year. The future value of the investment after one year is V_1 where:

$$V_1 = P(1 + y)$$

and after n years the value is V_n where:

$$V_n = P(1 + y)^n$$

If interest is compounded *semi-annually* then after n years:

$$V_n = P[1 + \frac{y}{2}]^{2n}$$

If interest is compounded m *times per year* then after n years:

$$V_n = P[1 + \frac{y}{m}]^{m \times n}$$

As the compounding interval gets smaller and smaller, i.e. as $m \to \infty$, the accumulated value after n years increases, because interest is being earned on interest. If interest is compounded continuously at rate y, then after n years the accumulated value is:

$$V_n = \lim_{m \to \infty} P[1 + \frac{y}{m}]^{m \times n}$$

Mathematicians have shown that this limit can be expressed in a simple way. In particular,

$$\lim_{m \to \infty} [1 + \frac{y}{m}]^{m \times n} = e^{yn}$$

where e^x is the exponential function that can be written as follows:[1]

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \text{.........for all values of x.}$$

Hence, with continuous compounding, the future value is:

$$V_n = Pe^{yn}$$

Example

The future value of a \$100 investment compounded at 10% per year simple interest is \$110; compounded semiannually the future value is $100(1.05)^2 =$ \$110.25; and compounded continuously is $100e^{0.10} =$ \$110.52.

Given one method of computing interest, it is possible to find another compounding rate that leads to the same terminal wealth. For example, assume the semi-annual compounding rate is y. Then after n years we have:

[1] The exponential expansion shows that when x is very small, $e^x \approx 1 + x$. In this case x is a simple return. For larger values of x, the higher order terms become important.

$$V_n = P[1 + \frac{y}{2}]^{2n}$$

The continuous compounding rate that leads to the same terminal wealth can be established by solving the equation for y^*:

$$V_n = P[1 + \frac{y}{2}]^{2n} = Pe^{y^* n}$$

Taking logarithms on both sides leads to

$$\begin{aligned} y^* n &= \ln[(1 + \frac{y}{2})^{2n}] \\ &= 2n \ln[(1 + \frac{y}{2})] \end{aligned}$$

or

$$y^* = 2 \ln[(1 + \frac{y}{2})]$$

Example

A semiannual rate of 10% per year is given. The equivalent continuously compounded yield is $y^* = 2\ln[(1 + \frac{y}{2})] = 2\ln(1.05) = 9.758\%$.

1.2 ANNUALIZING HOLDING PERIOD RETURNS

By convention market participants annualize an interest rate before quoting it to others, regardless of the compounding convention.

The price of a contract that promises to pay \$100 in 0.25 years is \$98.0. Let R represent the return obtained over the period. The holding period yield is

$$R = \frac{100 - 98}{98} = 0.0204 \text{ or } 2.04\%.$$

There are two common ways to annualize this return. The simplest method is to multiply the return by the number of holding periods in the year.

Example

The annualized simple interest in the last example is given by multiplying the holding period yield by the number of periods in the year, namely 4. Specifically, the annualized yield is $4 \times 2.04 = 8.16\%$

This method of annualizing goes under different names depending on the market. A six month yield annualized by multiplying by 2 is called a *Bond Equivalent Yield* or BEY.A one month yield annualized by multiplying by 12 is called an *Annualized Percentage Rate* or APR.

An alternative way to annualize a yield is by assuming compounding of the investment.

Example

The compounded rate of return in the last example is given by$(1+R)^n - 1$, where $n = 4$. This value is $(1.0204)^4 - 1 = 8.42\%$.

In the above example the compounding interval was taken to be quarterly. In many cases the investment period could be quite small, for example one day. In this case the compounded annualized return is $(1 + R)^{365} - 1$, where R is the one day return. If the holding period is small, then the calculation of annualized return can be approximated by continuous compounding. Specifically, for R close to zero, and n large, $(1 + R)^n \approx e^{nR}$.

Example

An investment offers a daily rate of return of 0.00025. A one million dollar investment for one day grows to $(1,000,000)(1.00025) = \$1,000,250$. The annual rate, approximated by continuous compounding, is $e^{365(0.00025)} - 1 = 9.554\%$

Given the annualized continuously compounded return is $y = 0.09554$, the simple return for a quarter of a year is $e^{(0.09554)(0.25)} - 1 = 2.417\%$.

The yield obtained by compounding is often referred to as the *Effective Annualized Rate* or EAR.

In all calculations care must be taken that the annual interest rate used is consistent in all calculations. For example, if a security returns 10% over a six month period, then the equivalent continuous compounded return is obtained by solving the equation $e^{y(0.5)} = 1.10$. Specifically, $y = \log(1.10)/0.5 = 19.06\%$

Compounding Over Fractional Periods

The future value of $\$P$ over 2 years when compounding is semi annual is $P(1+\frac{y}{2})^4$. Raising $(1+\frac{y}{2})$ to the power of 4 reflects four semiannual interest payments. If the time horizon is not a multiple of six months, then establishing the future value is a problem. For example, if the time horizon is 2.25 years, the future value could be written as $P(1+\frac{y}{2})^4(1+\frac{y}{2})^{0.5}$. The handling of the fractional period is not altogether satisfactory, and there is no real theory to justify this calculation. However, this calculation is one popular market convention.

Of course, if compounding was done quarterly, then the future value for the above problem is $P(1+\frac{y}{4})^9$. However, if the time horizon was 2.26 years, then fractional periods will still result and the compounding mechanism would then be unsatisfactory. If compounding is done continuously then the problem of handling fractional periods disappears. The future value of P dollars over T years is Pe^{yT}.

1.3 DISCOUNTING

The present value of one dollar that is received after n years, assuming the discount rate is $y\%$ per year with annual compounding, is given by

$$PV = 1 \times \frac{1}{(1+y)^n}$$

If compounding is done m times per year, the present value is:

$$PV = 1 \times \frac{1}{(1+\frac{y}{m})^{n\times m}}$$

If the one dollar is discounted continuously at the rate of $100y\%$ per year, the present value is:

$$PV = 1 \times e^{-y \times n}$$

1.4 BOND PRICES AND YIELD -TO- MATURITY

A coupon bond is a bond that pays fixed cash flows for a fixed number of periods, n say. Typically, the cash flows in all the periods are equal. At the last period a balloon payment, referred to as the *face value* of the bond, is also paid out. Typically, the coupon is expressed as a fraction of the face value of the bond. In what follows we will take c to be the coupon rate, and $C = c \times F$ to be the dollar coupon.

If the coupons are annual coupons, of size C, and the face value is F, then the *yield-to-maturity* of the bond is the discount rate, y, that makes the following equation true.

$$B_0 = \frac{C}{1+y} + \frac{C}{(1+y)^2} + ... + \frac{C+F}{(1+y)^n}$$

where B_0 is the actual market price of the bond.

The coupon of a bond refers to the dollar payout that is made in each year. If coupons are paid annually then each cash flow is of C dollars. Payments at frequencies of once a year are appropriate for typical bonds that are traded in the Eurobond market. For bonds issued in the US, however, the typical convention is for coupon payments to be made semiannually. Such a bond would therefore pay half its coupon payment every six months. In this case, the yield-to-maturity of a bond that matures in exactly n years, is the value for y that solves the following equation:

$$B_0 = \frac{C/2}{1+y/2} + \frac{C/2}{(1+y/2)^2} + ... + \frac{C/2+F}{(1+y/2)^{2\times n}} \tag{1.1}$$

Example

Consider a bond with a 10% coupon rate and 10 years to maturity. Assume the face value is \$100 and its price is \$102. The bond will pay 20 coupons of \$5.0 each, plus the face value of \$100 at the end of 10 years. The value of y that solves the above equation is given by $y = 9.6834\%$.

Clearly, the yield-to-maturity of a bond that pays coupons semiannually is not directly comparable to the yield-to-maturity of a bond that pays coupons annually, since the compounding intervals are different.

1.5 ANNUITIES AND PERPETUITIES

An annuity pays the holder money periodically according to a given schedule. A perpetuity pays a fixed sum periodically forever. Suppose C dollars are paid every period, and suppose the per period interest rate is y. Then the value of the perpetuity is:

$$P_0 = \sum_{i=1}^{\infty} \frac{C}{(1+y)^i}$$

The terms in the sum represent a geometric series and there is a standard formula for this sum. In particular, it can be shown that[2]

$$P_0 = \sum_{i=1}^{\infty} \frac{C}{(1+y)^i} = \frac{C}{y} \tag{1.2}$$

As an example, if a perpetuity paid out \$100 each year and interest rates were 10% per year, then the perpetuity is worth $100/0.10 = \$1000$.

The value of a *deferred perpetuity* that starts in n years time, with a first cash flow in year $n+1$, is given by the present value of a perpetuity or

$$P_n = \left(\frac{1}{(1+y)^n}\right)\frac{C}{y} \tag{1.3}$$

[2]To see this let $a = \frac{1}{(1+y)}$. Let S_n be the sum of the first n terms of the cash flows of the perpetuity. That is

$$S_n = aC + a^2C + ... + a^nC$$

Now, multiply both sides of the equation by a to yield

$$aS_n = a^2C + ... + a^nC + a^{n+1}C.$$

Subtracting the equations lead to

$$(1-a)S_n = aC - a^{n+1}C$$

Hence $S_n = \frac{aC - a^{n+1}C}{1-a}$. Substituting for a and letting $n \to \infty$ leads to $limit_{n\to\infty} S_n = \frac{C}{y}$.

By buying a perpetuity and simultaneously selling a deferred perpetuity that starts in n years time, permits the investor to receive n cash flows over the next n consecutive years. This pattern of cash flows is called an n-period fixed annuity. The value of this annuity, A_0 say, is clearly:

$$A_0 = P_0 - P_n = \frac{C}{y}[1 - \frac{1}{(1+y)^n}] \tag{1.4}$$

Rewriting the Bond Pricing Equation

A coupon bond with n annual payments $\$C$ and face value $\$F$ can be viewed as an n period annuity together with a terminal balloon payment equal to F. The value of a bond can therefore be expressed as

$$B_0 = \frac{C}{y}[1 - \frac{1}{(1+y)^n}] + \frac{F}{(1+y)^n} \tag{1.5}$$

where y is the per period yield-to-maturity of the bond.

When $F = \$1.0$, the coupon is given by $C = c \times 1 = c$. If $y = c$ then from the above equation, it can be seen that $B_0 = 1$. Hence, when the coupon is set at the yield to maturity, the price of a bond will equal its face value. Such a bond is said to trade at *par*. If the coupon is above (below) the yield-to-maturity, then the bond price will be set above (below) the face value. Such bonds are referred to as premium (discounted) bonds.

Unequal Intervals Between Cash Flows

So far we have assummed that the time between consecutive cash flows is equal. For example, viewed from a coupon date, the yield to maturity of a bond with semi annual cash flows is linked to its market price by the bond pricing equation:

$$B_0 = \sum_{j=1}^{m} \frac{C/2}{(1+y/2)^j} + \frac{F}{(1+y/2)^m}$$

where y is the annual yield to maturity, C is the annual coupon and m is the number of coupon payouts remaining to maturity. In this equation, the first coupon is paid out at date 1, in six months time. If the first of the m cash flows occurred at date 0, then the price of the bond is:

$$B_0 = \sum_{j=1}^{m} \frac{C/2}{(1+y/2)^{j-1}} + \frac{F}{(1+y/2)^{m-1}}$$

If the first coupon date is not immediate but occurs before 6 months, then the above equation must be modified. Specifically, the above equation can be used to price all the cash flows from the first cash flow date. This value, is then discounted to the present date. Specifically, the yield-to-maturity of a coupon bond is *defined* to be the value of y that solves the equation:

$$B_0 = \frac{1}{(1+y/2)^p}\left[\sum_{j=1}^{m}\frac{C/2}{(1+y/2)^{j-1}} + \frac{F}{(1+y/2)^{m-1}}\right] \tag{1.6}$$

where $p = t_n/t_b$ and t_n is the number of days from the settlement date to the *next* coupon payment, and t_b is the number of days *between* the last coupon date and the next coupon date. In this equation we have assumed that the total number of coupons to be paid is m. This way of handling fractional periods is the market convention used in the US Treasury bond market.

1.6 PRICE QUOTATIONS AND ACCRUED INTEREST

If a coupon bond is sold midway between coupon dates, then the buyer has to compensate the seller for half of the next coupon payment. In general, for Treasury bonds, the *accrued interest,* AI, that must be paid to the previous owner of the bond is determined by a straight line interpolation based on the fraction of time between coupon dates that the bond has been held. Specifically,

$$AI = \frac{t_l}{t_b}$$

where t_l is the time in days since the last coupon date, and t_b is the time between the last and next coupon date. The computation of accrued interest using this convention is termed "actual/actual". The first actual refers to the fact that the actual days betwen coupons are used in the calculation. The second actual refers to the fact that the actual number of days in a year are used. The above convention is standard for Treasury bonds traded in the US. Other methods of computing accrued interest that apply in different markets will be considered later.

Market convention requires that US Treasury bond price quotations be reported in a particular way. A face value of $100 is assumed and the quotation ignores the accrued interest. The actual cost, or *invoice price* of a bond, corresponding to B_0 in the equation (1.6) given a quotation is:

$$\text{Invoice Price} = \text{Quoted Price} + \text{Accrued Interst}$$

The specific rule for computing accrued interest and translating quoted prices from a newspaper into market prices vary according to the particular fixed income product.

Example

A Eurobond is a bond issued by a non European firm in Europe. Typically, interest is paid annually, and yields are simple annual yields. The accrued interest in this market are not based on "actual/actual", but rather on "30/360". In this convention each month is counted as having 30 days and each year has 360 days. Say a bond pays coupons on August 1st of each year and the settlement date for the transaction falls on April 10th. The seller has held the bond for 8 months and 10 days. Under this convention the accrued interest is based on $\frac{8\times 30+10}{360} = \frac{250}{360}$. Specifically, the accrued interest is $25/36\ ^{th}$ of the annual coupon. This accrued interest is added to the quoted price to obtain an invoice price. Given the invoice price, a yield for this product can be obtained using the appropriate bond pricing equation.

Specific products and the market conventions related to compounding frequency, quotation format, and the handling of accrued interest will be discussed in more detail in future chapters. The important point here is that the conventions are market specific.

The accrued interest convention makes the quoted price process smooth over time. Actual market prices of bonds fall at coupon dates. Just before a coupon, the price of a bond with n years to maturity is

$$B_0^- = \frac{C}{2} + \sum_{i=1}^{2n} \frac{C/2}{(1+y/2)^i} + \frac{100}{(1+y/2)^{2n}}$$

Since the seller has held the bond over the entire period, $(t_l = t_b)$ the accrued interest is $\frac{C}{2}$ and the quoted price, Q_0^- say, is the above market price less $\frac{C}{2}$, or

$$Q_0^- = \sum_{i=1}^{2n} \frac{C/2}{(1+y/2)^i} + \frac{100}{(1+y/2)^{2n}}$$

Immediately after the coupon has been paid, the bond price is given by

$$B_0^+ = \sum_{i=1}^{2n} \frac{C/2}{(1+y/2)^i} + \frac{100}{(1+y/2)^{2n}}$$

The drop in price, $B_0^+ - B_0^-$, equals the actual coupon paid out. Since the new accrued interest is now zero ($t_l = 0$), the new quoted price equals the new market price, which in turn equals the old quoted price. That is, quoted prices remain unchanged.

1.7 COMMON INTEREST RATE CONVENTIONS

Securities are issued with cash flows that occur at different time intervals. To compare rates it is often necessary to switch from one type of rate, based on a particular compounding interval, to another rate.

Example

A rate of 9% semi-annual is equivalent to a $(1 + \frac{0.09}{2})^2 - 1 = 9.2025\%$ annual rate.

A 9% semi-annual rate is also equivalent to a daily rate of $(1.092025)^{\frac{1}{365}} - 1 = 0.024122\%$ per day. On annualizing this rate we obtain $0.024122 \times 365 = 8.80445$.

A 9% semi-annual rate is equivalent to a daily rate of 0.02412%. Over a 100 day period, the rate is $(1.0002412)^{100} - 1 = 0.0244102$ or 2.44102%. Annualizing this rate we obtain $2.44102 \times \frac{365}{100} = 8.9097\%$. The effective annualized rate of this loan for 100 days is 8.9097%.

Table 1.1 shows the market convention of rates in particular markets.

Table 1.1 Market Convention of Rates in Particular Markets

UK Money Markets	Annual Actual/365
US & Euromoney Markets	Annual Actual/360
US Treasury Bonds	Semi-annual Actual/365
Eurobonds	Annual 30/360
US Federal Agencies, Municipals, Corporates	Semi-annual 30/360
US Commercial Paper, Bankers Acceptances	Discount Basis, Actual/360
Commercial Paper	Discount Basis, Actual/365

Examples

(i) Assume the semiannual coupon periods are divided into 181 and 184 days. Assume 10m dollars are borrowed at 10 % semiannual actual/365. Then, the coupon payments of \$1m over the year would be split up into payments of $10m \times 0.10 \times \frac{181}{365} = \$495,890.41$, and $10m \times 0.10 \times \frac{184}{365} = \$504,109.58$.

(ii) The same loan done on a 30/360 basis would have two cash flows of \$500,000 each. The annual total is the same, but the size and timing of the individual cash flows are different.

(iii) Table 1.2 shows the effective annual rates of a 10% quotation for several market conventions.

Table 1.2 Examples of Market Conventions

Convention	Computation	Effective Annual Rate
Annual Actual/365	$(1 + 0.10) - 1$	=10.0%
Annual Actual/360	$(1 + 0.10\frac{365}{360}) - 1$	=10.14%
Semi-annual Actual/365	$(1 + \frac{0.10}{2})^2 - 1$	=10.25%
Semi-annual Actual/360	$(1 + \frac{0.10}{2}\frac{365}{360})^2 - 1$	=10.40%
Monthly Actual/365	$(1 + \frac{0.10}{12})^{12} - 1$	=10.47%
Monthly Actual/360	$(1 + \frac{0.10}{12}\frac{365}{360})^{12} - 1$	=10.62%

1.8 YIELDS AS A METHOD OF COMMUNICATING PRICES

The invoice price of a bond is the amount of dollars one requires in order to purchase it. Once you know the price, you can compute its *yield* using an appropriate formula. Conversely, if the yield of a bond is given, then provided you understand the market convention associated with the fixed income product, the unique price of the bond can be established, the accrued interest computed, and a quoted price can be established. The mapping from yields to quoted prices requires understanding the compounding mechanism (eg. annual or semiannual), the handling of fractional periods and the computation mechanism for accrued interest. Given these rules, prices can be quoted in yield form. While yields associated with different fixed income products may be useful for communicating price information, one has to be careful in interpreting these numbers. Higher yields do not necessarily imply higher returns, or higher risks. As a simple example, comparing yields of a coupon bond that pays annually, with a coupon bond that pays semiannually may be misleading. While in some cases the yield of a fixed income product may have a simple economic interpretation, in others no simple interpretation exists. For example, consider a straight default free coupon bond. Its price is the present value of the bonds cash flows using the yield as a discount rate. On the other hand, consider a coupon bond that has a call feature. The yield that is given to characterize its price cannot be interpreted as a discount rate for all the promised cash flows to the maturity date.[3]

In general, then, while "yields" are often used to characterize prices of fixed income products, in general they may not have simple economic interpretations, and certainly do not provide a common ground by which their relative benefits can be accessed.

Given a bond price, there is no theoretical reason why coupon bonds have to have their yields to maturity computed according to any market convention. For example, we could define the continuously compounded yield to maturity of a bond that has face value F, and pays C dollars at times t_1, t_2,....,t_n and face value F, is given by the value y that solves the equation.

$$B = Ce^{-yt_1} + Ce^{-yt_2} + Ce^{-yt_3} + ... + (F + C)e^{yt_n} \tag{1.7}$$

In this equation the times t_1, t_2..., t_n are all expressed in years and need not be equidistant. This definition of a yield to maturity is as valid as any other definition, but is not adopted in any specific market as a normal market convention.

[3]We shall explore this in another chapter. The problem for callable bonds is that the exact number of future cash flows is not certain since the bond can be called at any time after the call date.

1.9 CONCLUSION

The purpose of this chapter has been to review the basics of discounting at annual, semiannual and continuously compounded rates and to obtain some insight into how prices are connected to specific yields according to market conventions. In order to obtain the invoice price of a bond, its quoted price may have to be adjusted by accrued interest. The computation of accrued interest varies according to the particular product. We illustrated the adjustment for Treasury bonds, where the "actual/actual" rule holds and for eurobonds, where a "30/360" rule holds. Given the invoice price, the quoted yield for the particular fixed income product can also be obtained. The way in which the yield is computed also depends on the particular product. Treasury bond yields, for example, are reported in semiannual form while Eurobonds are reported using annual compounding. Given the market convention, price information can be conveyed using their appropriate yields. In general, however, the particular yield-to-maturity statistic that is computed for a product may not provide useful economic information relating to its potential return or risk.

1.10 EXERCISES

1. An investment requires an initial investment of \$100 and guarantees \$104 back in 0.25 years.
   (a) Compute the holding period return.
   (b) Compute the simple annualized yield.
   (c) Compute the compounded annualized yield for the investment, assuming quarterly compounding.
   (d) What is the annualized continuously compounded rate of return for this investment.

2. A discount bond with a maturity of 5 years and a face value of \$1000 is priced at \$670.03.
   (a) Compute the continuously compounded yield -to-maturity.
   (b) Compute the semi annualized yield to maturity.

3. A discount bond with a face value of \$1000 is currently priced at \$786.60. The maturity of the bond is 6 years. The bond, however, is callable in 3 years for a price of \$860.71.
   (a) Compute the continuously compounded yield-to-maturity.
   (b) The yield to call is the yield to maturity obtained under the assumption that the call date is the maturity date. Compute the continuously compounded yield-to-call.
   (c) Interpret the above two numbers and comment on the potential problems with interpreting these two yield measures.

4. The quoted (sometimes called the clean or flat) price of a Treasury bond with settlement date January 6^{th}1999 is \$100.09375. The bond's coupon is 4 1/4. It matures on November 15^{th}2003. The number of days in the current coupon period is 182, and the number of days from settlement to the next coupon date is 130 days. Compute the accrued interest and the invoice price (sometimes called the full or dirty price) of the bond. What happens to the dirty price at the coupon date.

5. In this problem you will learn how to use excel to compute prices of coupon bonds when cash flows are equally spaced. In particular, you will compute bond prices four different ways. The main idea here is to show that the analytical solution for the bond price is helpful, and to introduce you to excel's PRICE function that produces a clean price and is fairly useful. The benchmark model we will solve is a 5 year maturity bond paying annual coupons rate of 5% seminanually. The face value is

\$100. The yield-to-maturity is given as 6%. In excel set the inputs up as follows:

INPUTS	
Annual Coupon Rate (AC)	0.05
Yield to maturity (Y)	0.06
Number of payments per year (num)	2
Number of Periods (N)	10
Face Value (FV)	100

OUTPUTS	
Discount rate/Period (Rate)	0.03
Coupon payment (c)	2.5

For each of these variables label them using Insert,Name,Define. Excel will now recognise these variables when you refer to them. Now we are ready to compute bond prices.

(a) Set up 11 colums numbered 0 to 10. These refer to the time periods. There will be three rows under these columns. The first row is called time (in years). For this problem it will be the period number divided by 2. The second row will contain the cash flows. For this problem it will be a row of 2.5 dollars starting from period 1 and ending in period 9. In period 10 there will be a cash flow of 102.5. The final row will then contain the present value of each of these cash flows. The bond price is then obtained by adding these numbers up. Confirm that you obtain a value of 95.7349. Note that if we change the number of periods, we will have to add more columns in our spreadsheet. So this method is not very useful.

(b) Now repeat the exercise of pricing this bond, but this time use the analytical formula for bond pricing. So in one equation, using the variable names, you can obtain the price. This formula has an advantage over (a) in that the number of periods can be changed and the price will automatically update.

(c)Now compute the bond price using the PV function in excel. This function requires the Rate (Rate), Number of periods (N), coup (c), and face value, (FV), as inputs.

(d) Finally, compute the bond price using the PRICE function in excel. This function requires the settlement date, the maturity date, annual coupon rate, yield-to-maturity, face value and the number of payments. To use it for an example make up a settle date (eg 01/01/2000) and then add 5 years to get the maturity date. To do this use the excel DATE command eg DATE(2000+5,1,1). This will give you a maturity date exactly 5 years later. In general the PRICE function gives you a quoted, flat, or clean price. The actual invoice, full, or dirty price is obtained by adding on the accrued interest. In this above problem there is no accrued interest so the clean and dirty prices are equal. We will use the PRICE function in the next chapter.

1.11 APPENDIX

Here are the details of some Calendar Conventions:

(1) Actual/Actual Calendar

Suppose the coupon bond pays coupons every 6 months, and that the settlement date is between coupons.

- Price the bond as if you were at the next coupon date. Make sure to add in the coupon that is received on that date.
- Present value this figure using the bond pricing equation. The fractional period is determined based on the number of days.

$$p = \frac{t_n}{t_b}$$

The discount factor is

$$\frac{1}{(1 + y/2)^p}$$

(2) Actual/Actual Calendar (Act/365f)

This market convention is the same as the above except every year (including leap years) is assumed to have 365 days.

(3) 30E/360 Calendar or EuroBondBasis

Let date 1 be the early date and date 2 the late date. Let Y_i, M_i and D_i be the year, month, day date for $i = 1, 2$.

Then

- $d_1 = 30$ if $D_1 = 31$
- $d_1 = 30$ if $D_1 \geq 28$ and $M_1 = 2$ and date 1 is a coupon date.
- $d_1 = D_1$ otherwise. Let
- $d_2 = 30$ if $D_2 = 31$ and $D_1 \geq 30$
- $d_2 = D_2$ otherwise.
- Then, the number of days between date 1 and 2 is:

$$n_{days} = 360(Y_2 - Y_1) + 30(M_2 - M_1) + (d_2 - d_1)$$

Now:

- Compute the days in 30/360 form between settlement to next coupon, t_n.
- Compute the days in 30/360 form between coupons, (last coupon to next), t_b.
- Compute $p = \frac{t_n}{t_b}$.
- Use the bond pricing equation.

Corporate, Municipal and Federal Agency bonds use this method.

(4) 30/360 Calendar or BondBasis

This is similar to 30E/360 except that if interest falls on the 31st of a month, it is moved forward to the beginning of the next month for purposes of day calculation. Actually, it is not moved forward if the beginning interest period falls on the 30th or 31st. In this case, the interest period is moved from the 31st to the 30th.

Then

- $d_1 = 30$ if $D_1 = 31$
- $d_1 = 30$ if $D_1 \geq 28$ and $M_1 = 2$ and date 1 is a coupon date.
- $d_1 = D_1$ otherwise. Let
- $d_2 = 1$ and M_2 is increased by one if $D_2 = 31$ and $D_1 \geq 30$
- $d_2 = D_2$ otherwise.

(5) Actual/360 Calendar

- Compute the actual days between settlement to maturity, $t_{s,m}$.
- Compute the actual days between issue to settlement, $t_{i,s}$.
- Compute the actual days between issue to maturity, $t_{i,m}$.
- Redefine these numbers by dividing them by 360.

For a CD with coupon c, quoted in annual terms, the interest payment is:

$$100 \times c \times \frac{t_{i,m}}{360}$$

Given a simple interest quote of y, the full dollar price of a \$100 par amount of a CD with coupon rate c is

$$B = 100 \times \frac{1 + c(t_{i,m}/360)}{1 + y(t_{s,m}/360)}$$

The accrued interest is

$$100 \times c \times \frac{t_{i,s}}{360}$$

and the flat price is obtained by subtracting the accrued interest from the full price.

If the start day, D_1 is $T_1/M_1/Y_1$ and the end day, D_2 is $T_2/M_2/Y_2$, the time between the two dates can be expressed in formulae:

Act/Act: $Y_2 - Y_1 + \frac{D_2 - Date(Y_2)}{Date(Y_2+1) - Date(Y_2)} - \frac{D_1 - Date(Y_1)}{Date(Y_1+1) - Date(Y_1)}$

Act/365f: $\frac{D_2 - D_1}{365}$

30E/360: $Y_2 - Y_1 + \frac{M_2 - M_1}{12} + \frac{min(T_2,30) - min(T_1,30)}{360}$

30/360: $Y_2 - Y_1 + \frac{M_2 - M_1}{12} + \frac{T_2 - min(T_1,30) - max(T_2 - 30,0) * \delta_F}{360}$

Act/360: $\frac{D_2 - D_1}{360}$

The function Date delivers a numerical value for that date, initialized to zero at some long distant past date, and incremented by one for each successive day. δ_F is 1 if $T_1 = 29$ and 0 otherwise.

The computation of time periods using different conventions is illustrated below. We take a zero coupon bond with face value 100 that matures on

December 31^st 2004, and is valued at 96.543 on February 14^{th} of the same year. The table shows the different measures of time based on the convention. The table also shows three different annualized yields. The rates are defined by the values y for which $B = 0.96543$.

Simple $\quad B = \frac{1}{1+y(T-t)}$

Discrete $\quad B = (\frac{1}{1+y})^{-(T-t)}$

Continuous $\quad B = e^{-y(T-t)}$

We have:

	Days	Years	Simple	Discrete	Continuous
Act/Act:	321	0.87704918	4.08	4.09	4.01
Act/365f:	321	0.879452055	4.07	4.08	4.00
Act/360:	321	0.891666667	3.88	4.02	3.95
30/360:	317	0.880555556	3.93	4.08	3.99
30/E360:	316	0.877777778	3.94	4.09	4.01

This example clearly illustrates that while the bond price is 96.543, the yield can range from 3.88 to 4.09 depending on the day count convention and depending on how the rate is computed. Ultimately, yields are market conventions that can be rather confusing to learn. One price can map into many different yields. Ultimately, it makes little difference on which yield mechanism is used, as long as the user understands how the yield can be backed up into an unambiguous price.