Competition and Diversification Effects in Supply Chains with Credit Risk*†

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Abstract

We study the effects of credit risk in a supply chain where one retailer deals with competing risky suppliers who may default during their production lead-times. The suppliers, who compete for business with the retailer by establishing wholesale prices, are leaders in a Stackelberg game with the retailer. The retailer, facing uncertain future demand, chooses order quantities while weighing the benefits of procuring from the cheapest supplier against the advantages of reducing credit risk through diversification. If the wholesale prices were exogenous, the retailer would benefit by choosing suppliers that had low default correlations. However, when prices are endogenous, low supplier default correlations dampens competition among the suppliers, increasing the equilibrium wholesale prices. We show that the retailer prefers suppliers with highly correlated default events. In contrast, the suppliers and the channel prefer defaults that are negatively correlated.

1 Introduction

This paper is concerned with the problems faced by a retailer who deals with competing risky suppliers who may default on their obligations to deliver order quantities at the end of a given production lead time. Random supply shocks could arise from failures, disruptions or strikes in one or more of the suppliers' plants or from financial defaults as firms enter financial distress which effects their operations. To address this problem the retailer might consider diversifying order quantities among competing suppliers. If the wholesale prices provided by the suppliers are taken as *exogenous*, then the retailer is faced with a classical portfolio problem, where the default risks can be optimally diversified by splitting orders among multiple suppliers. The benefits of diversification increase as the correlation between the different supplier defaults decreases. If, however, the wholesale prices charged by the suppliers are *endogenous* then the benefits from diversification for the retailer may depend on the actions of suppliers. In this case, the analogy with a classical portfolio selection problem is no longer valid and the analysis requires game theory tools rather than simple portfolio optimization tools.

The basic idea of the paper is that a retailer, facing random supply shocks, faces a tradeoff between *diversification* and *price competition* effects. When suppliers compete for business from the retailer, intuition suggests that the degree of price competition might depend on the correlation of their defaults. For example, consider two suppliers identical in all aspects except default correlation. If their default processes were perfectly positively correlated, then, from the retailer's perspective, the two suppliers are identical and from the typical portfolio viewpoint, there is no diversification benefit. However, in this case, since the goods are completely substitutable, one might expect fierce price competition between the suppliers. Indeed, the competition from the loser holds down the price the winner can charge, and this beneficial competition increases the retailer's profits. This increase in profits, however, has no benefit to the supply chain as a whole; it is pure rent extraction from the suppliers. Now, as correlation of the supply shocks decreases, the usual portfolio viewpoint would indicate that diversification benefits would accrue. However, as correlation decreases, the goods become less than perfect substitutes, and this reduces the price competition, and, in equilibrium, allows room for the suppliers to profit by charging higher prices. Indeed, in the extreme case where supply shocks are perfectly negatively correlated and the diversification benefits are the highest, the two suppliers do not coexist in the same probabilistic states of nature, have no need to compete over prices for business from the retailer, and, acting as monopolists, they can charge the retailer up to the full marginal value of the additional unit.

This paper carefully explores the tradeoffs that exist between competition and diversification effects in the supply chain, and identifies the strategies and pricing policies that result in equilibrium. By pressing beyond the standard portfolio selection problem (with a single decision maker), where the role of correlation has been well studied, and tackling the endogenous nature of pricing policies (with multiple decision makers) and, consequently, competition, we hope to learn more about the true costs and benefits of retailers who may be inadvertently reducing competition by diversifying orders among several suppliers.

While financial default risk is just once source of possible supply chain disruptions, its impact can be huge. Barton, Newell and Wilson (2003), for example, share the following experience with supplier defaults:

"In 1997, one South Korean automaker saw many of its parts suppliers go under. Without backup suppliers, it couldn't increase production for export when the won was devalued. While foreign distributors begged for more cars to sell, production lines were idle back at home for lack of critical parts. The company weathered the storm but never fully recovered its market position and was eventually acquired by another domestic automaker".

The default of a supplier need not be as catastrophic as the above or as WorldCom's default to have a significant impact on the firm's bottom line. With the wide acceptance of JIT manufacturing any disruption in supply could have serious ramifications for the firm.[1]

Recognition of supply disruptions and default risk among counterparties in a supply chain is now more important than ever before. As banks have tightened their credit policies, firms have found it more difficult to raise funds, and this has created a need for retailers and suppliers to work closely together to better bundle products with loans. As a result, it is not surprising that this form of trade credit is now by far the largest source of short-term debt financing for firms, representing over one-third of the current liabilities of all non-financial corporations. As a result of deteriorating credits, retailers are now more inclined to split orders among several suppliers and to diversify their customer base. The impact of diversification may, however, come at a cost, and our paper explores the effects of these costs.

The main theme of this paper can be intuitively described by the following stripped down

[1] In addition to the direct costs of supplier bankruptcy, a firm could also suffer from indirect costs. For example, a supplier, in bankruptcy or experiencing financial distress, could struggle to retain qualified workers. The managers of such a firm have incentives to cut costs by eliminating "non-essential" activities, such as quality control, R&D investments, etc. Consequently, the deteriorating quality of its goods may ultimately lead to higher costs for the retailer.

example. Suppose we have a demand for the retail product of one unit. Assume two identical suppliers, 1 and 2, offered the good to the retailer, and that each supplier has a 50% chance of defaulting on delivery of the good. Each supplier's cost of production equals c and the retail price equals s. The risk-free rate is zero, and the total production cost for both suppliers, 2c, is less than s. This last assumption is required so that we do not have to worry about a zero order being optimal.

First, suppose the defaults of the two suppliers are perfectly correlated. In this case, the goods of the two suppliers are perfect substitutes from the point of view of the retailer. Bertrand competition between suppliers forces each supplier's profit to be zero, and in equilibrium the wholesale price, K say, equals c.

Now consider the case where defaults have perfect negative correlation. In this case the goods are no longer substitutes. Each supplier is now a monopolist making a first and final offer for supply for a section of the probability space. These monopolists can each charge the retailer the total value of the good to the retailer, which equals the probability of delivery times the retail price, ie $0.5s$. Thus the two suppliers, together, extract all the surplus from the supply chain.

Finally, consider independent defaults. First, consider the case when the retailer optimally buys two units. In this case, the marginal value of buying a second unit, given that the retailer has already bought a first unit equals the probability that the new supplier will survive and the first supplier will default, which is 0.25, times the retail value of the good, s. That is $0.25s$. If the retailer is buying two units, the suppliers must be charging this marginal value. Thus, in any equilibrium in which the retailer buys 2 units, both suppliers charge $0.25s$. Such an equilibrium exists whenever $c \leq s$. Next, consider the case when the retailer optimally buys one unit. In this case the excluded supplier has to earn zero profit from undercutting the favored supplier and thus price must equal cost. Such an equilibrium will exist only if the retailer, at the equilibrium prices, cannot deviate to buying one more unit. The cost, at equilibrium of one more unit is as before, namely $0.25s$. Thus the one unit purchase equilibrium exists whenever $c \geq 0.25s$, the complementary region of the parameter space to the two unit purchase equilibrium. In summary, the independent case works out like a Bertrand one unit case when the retailer orders one unit, pitting suppliers against each other, and works out like the two unit case when the retailer ensures product supply, at the cost of increasing supplier bargaining leverage. When profit margins are high, the two unit equilibrium results, and when margins are low, the one unit equilibrium results.

The above example, highlights the main issues that we deal with. Implicit in this example is the fact that demand was certain at one unit, that all agents had full information and were risk-neutral, that only two suppliers were available, and that the timing of payments, were up-front, rather than upon delivery. Implicit, also, was the assumption that the suppliers, while competing with each other, were collectively Stackelberg leaders in the game. In particular, the retailer responded to their pricing policies.

Throughout this paper we will assume that bargaining power does rest with the suppliers. If the reverse allocation of bargaining power was assumed, namely that the retailer could always fix the suppliers prices at their marginal values, then we would obtain uninteresting results, namely that the retailer could extract all the rents from the supply chain. Interestingly, what we find for our setup, where the suppliers have greater bargaining power, is that the retailer has a preference for positive correlations among the supplier default processes! That is, the price competition effects, induced by positive correlations, exceed the diversification benefits that arise from lower, or even negative correlations. Of course, what is best for the retailer, might be at odds with what is best for the supply chain as a whole. What we find is that negative correlations, rather than positive, improve the supply chain performance as a whole.

Our analysis also shows how supply shocks affects all agents in a supply chain. Not surprisingly, we find that increasing the intensities of supply shocks hurts all firms in the chain. However, the way in which the decline in profit is shared among the different agents in the supply chain, varies in specific ways that can be linked, ex ante, to the shape of the customer demand cumulative distribution function.

In addition to determining both direct and indirect effects of default correlation on performance of firms in a supply chain, this paper also examines the consequences of the suppliers offering different payment policies, ranging from up-front payments for the entire order quantity, to on-delivery payments where only the goods that are delivered are paid for. In the presence of supplier default risk the timing of the payments from retailer to suppliers is an important consideration.

The paper proceeds as follows. In section 2 we review the related literature on supply chains with disruptions. In section 3 we introduce our basic model, describe the default processes and the nature of competition. We investigate the effects of timing of the retailer-to-supplier payments and identify a class of pricing policies for which, in equilibrium, the suppliers and retailer are indifferent over the timing of payments. In section 4, we examine the model with only one supplier and establish conditions on the demand distribution function that identify the echelon of the supply chain that bears the majority of the supply risk. Our main results are provided in Section 5 where the analysis is extended to the 2- and n-supplier cases. The insights derived from the model and their implications on the strategic behavior of the firms are carefully examined.

2 Literature Review

Our problem relates to the random yield research, the majority of which is dedicated to finding optimal inventory and procurement decisions for a single firm whose supply in not certain. Yano and Lee (1995) offer an excellent review of this literature and propose a useful taxonomy. Our research can be linked to several of their categories. The retailer's problem in our model with one supplier is a "single period discrete time" random yield model with a stochastically proportional yield. Therefore, the retailer's ordering policies that we derive are similar to the policies obtained by Gerchak, Parlar and Vickson (1986). The retailer's problem in our model with two suppliers is similar to a single-period model by Anupindi and Akella (1993) and falls into the "multiple suppliers of the same item" category of Yano and Lee (1995). Anupindi and Akella (1993) study one- and multi- period discrete-time problems of a retailer who can order from one or two suppliers whose failure processes are uncorrelated. The authors derive optimal ordering policies under various stochastic yield assumptions including all-or-nothing, partial recovery, and delayed delivery. Our analysis generalizes their findings in that we consider suppliers with correlated default processes and with the wholesale prices determined endogenously.

The problem of a single supplier selling to a newsvendor has been addressed by Lariviere and Porteus (2001). The authors consider a one-period Stackelberg game with a single supplier, who announces a wholesale price, and a single retailer, who responds by choosing an order quantity. Under mild assumptions on the demand distribution, they prove the existence and the uniqueness of the solution to this game and provide conditions that the equilibrium order quantity must satisfy. The authors also study how market size and demand variability affect the equilibrium solution, the firms' profits, and the overall supply-chain performance. In our paper we add a possibility of supplier's default to the problem in Lariviere and Porteus (2001) and focus on the effects of the supply risk on the performance of the supply-chain. We further generalize the problem in Lariviere and Porteus (2001) by considering a game with more than one supplier.

A relatively recent area of research in the supply chain field is the design of reliable logistics distribution systems (usually global systems). See, for example, Vidal and Goetschalckx (1997), Vidal and Goetschalckx (2000), Snyder and Daskin (2003), Bundschuh, Klabjan and Thurston (2003) and references therein. The terrorist attack on September 11, 2001 prompted many companies to review their supply chains for potential vulnerabilities. Both academics and practitioners have published a number of articles addressing questions of managing supply chains under the threat of terrorism (see Sheffi (2001), Rice and Caniato (2003a), and Rice and Caniato (2003b)). Besides modeling methodology, the biggest difference between those papers and our work is our focus on the *strategic* interaction and games between suppliers and retailers and among suppliers.

One could also interpret the problem considered in this paper as a multi-supplier sourcing problem. Recent survey articles by Elmaghraby (2000) and Minner (2003) describe a variety of models proposed in a multi-supplier supply chain management literature. In the description of future research, Minner (2003) suggests that models with competing suppliers and inventory considerations due to the demand or lead time uncertainty have not been explored sufficiently yet. Our paper attempts to rectify this shortfall.

In the analysis that follows there are two fundamental sources of uncertainty. The first relates to the demand distribution for the good sold by the retailer. The second relates to the joint default process for the two suppliers and the random yields for the orders, should default occur. If we assume all agents are risk-neutral, then the true demand distribution has to be given exogenously and the true joint default process has to be estimated, typically from historical default data. Usually such data is limited and one has to use average values obtained from firms in similar industries. Rating agencies, for example, provide default correlations by industry. Examples of such studies include Carty (1997) and Erturk (2000).

Rather than estimate actual default probabilities, if the focus is on financial default events, it may be more appropriate to estimate risk-neutralized probabilities. Indeed, pricing models for defaultable claims, as developed by Merton (1974), Jarrow and Turnbull (1995), Duffie and Singleton (1999a), Lando (1998) and others, all require risk-neutralized processes rather than the true data-generating processes. If the suppliers are large firms that have traded equity, debt and perhaps other claims on the assets of their respective firms, then these prices contain information on the parameters of the default processes. For example, if the price of a supplier's debt falls, then this is a signal that default is more likely. The idea, then, is to use traded prices to infer parameter estimates for processes that control the well being of the firm.[2]

[2]The first family of models for defaultable claims, dating back to Merton (1974), are based on the structural notion that default occurs at the moment when the firm's assets drops below its liabilities. Extensions of these models to handle multiple defaults, primarily through modeling correlation among the equity values, has been considered by Hull and White (2001) and Zhou (1997). An alternative reduced form approach treats defaults as a jump process with an exogenous intensity process. Models in this family include Jarrow and Turnbull (1995), Duffie and Singleton (1999a) and many others. Such models are now routinely used to price credit derivatives on single firms. These models can be extended to incorporate default correlation in several ways. The first approach is to allow the default intensities to follow stochastic correlated processes. However, such approaches produce default correlations that are too small. Jarrow and Yu (2001) develop infection models, where the intensity of surviving firms are heavily influenced by recent defaults. Duffie and Singleton (1999b) present an alternative approach where point processes

3 Model Assumptions

Consider a model of a simple supply chain with one retailer and several suppliers, who produce perfectly substitutable products using technologies with identical production lead-times. Without loss of generality, assume that the lead time is 1 and that production begins at date 0 and ends at date 1. At date 0, the suppliers determine their pricing policies. The retailer responds by choosing order quantities. Thus, the suppliers compete with each other for the retailer's business, and collectively, they are Stackelberg leaders in a game with the retailer. As soon as the suppliers receive orders, they commence production. The per unit production cost for supplier i is c_i and the bulk of production costs is incurred up-front (at date 0).

At date 0 the retailer is faced with ordering decisions to satisfy uncertain demand, D, that is realized at date 1. The cumulative distribution function for demand, $G(\cdot)$, is continuous with probability density function $g(\cdot)$.

We assume that the time of the disruption for supplier i is a random stopping time which is unaffected by the pricing and payment policies and, in particular, by the order quantities. This assumption is justified if the default risk is attributed to exogenous events, such as strikes, or in the case of financial defaults, if the business that the retailer brings to the supplier is a small part of the supplier's full line of business activities. Further, to focus on the tradeoffs of diversification and competition we assume away all agency costs and consider a full information model. In particular, the joint default distribution is known by all agents.[3]

If a supplier defaults during the production cycle, the exact quantity delivered will depend on the timing of the default. In particular, if the default occurs early (late) in the production cycle then the random yield will be low (high). In general, let β_i be a random variable that represents the proportional random yield for the supplier i, with $0 \leq \beta_i \leq 1$.

The default and demand random variables are independent and the per unit retail sales price, s, is predetermined. One can think of s as the expected value of the future random price, $S(T)$, where $S(T)$ is independent from other random variables in the model. We assume, for simplicity, that any unsatisfied demand is lost and any unsold goods are costlessly discarded. Holding and shortage costs could be easily added to our model, however, because they do not alter the nature

are used to trigger simultaneous defaults. More recently, Schönbucher and Schubert (2001) permit individual firms to have arbitrary marginals, and then they build in a dependency structure via a copula function.

[3]In practice, this assumption might be severe. However, our goal is to identify the nature of the tradeoffs of diversification and competition effects without clouding the issues by incorporating additional realism. While agency effects can distort the results, it is doubtful that they will overturn the direction of our findings.

of our findings we omit them, again, for ease of exposition.

We assume that our agents are risk-neutral, and that the riskless interest rate is r. [4]

As will be shown in section 3.1, one can assume without loss of generality that the payments from the retailer to the suppliers are made at date 0. Then the problem of the retailer, who can place orders with N suppliers, is

$$\max_{z_1 \geq 0, z_2 \geq 0, \ldots, z_N \geq 0} \left\{ e^{-r} s \, E \left[\min(D, \sum_{i=1}^{N} \beta_i z_i) \right] - \sum_{i=1}^{N} K_i z_i \right\}, \tag{1}$$

where $\{K_i\}_{i=1}^N$ are the wholesale prices set by the suppliers. Denote by $z_i(K_1, ..., K_N)$ the retailer's order quantity to supplier i. The suppliers compete with each other for the retailer's business and solve the following optimization problems

$$\max_{K_i \geq 0} (K_i - c_i) \, z_i(K_1, ..., K_N), \quad i = 1, 2, ..., N. \tag{2}$$

3.1 The Timing of Payments for the Retailer's Orders

In the presence of supply risk, the timing of retailer-to-supplier payments is important. To reduce credit risk exposure, the retailer would prefer to pay at date 1, after the product has been delivered, whereas the suppliers would prefer to receive full payments at date 0, before production has begun. Many payment schemes exist. For example, a supplier may announce a policy of the form $\phi_i = \{\alpha_i, w_i^F, w_i^D\}$ where w_i^F is the per unit *up-front* wholesale price, w_i^D is the per unit *on-delivery* price, and $0 \leq \alpha_i \leq 1$ is the proportion of the units for which the retailer must pay up-front. Alternatively, a policy may call for the up-front payment of a certain percent of the total expected cost with the balance due on-delivery. Both of these policies are examples of linear pricing policies.

Let $P(z_1, z_2, ..., z_N)$ be the retailer's discounted expected revenue obtained from selling the product after orders of size z_i, $i = 1, ..., N$ are placed with the suppliers.

$$P(z_1, z_2, ..., z_N) = e^{-r} s E[\min(D, \sum_{i=1}^{N} \beta_i z_i)], \tag{3}$$

[4] An alternative assumption is that supply risk arise solely from firms defaulting. Further, if our suppliers have debt that is publicly traded, then in a complete arbitrage free market, the bond prices will reflect credit risk. In such a market, standard finance arguments guarantee the existence and the uniqueness of a pricing measure, also called risk-neutral measure [see, for example, Harrison and Kreps (1979), Harrison and Pliska (1981)] under which asset prices, normalized by the money fund that grows at the riskless rate r, are martingales. In such an economy, regardless of preferences, each firm in a supply chain should maximize its expected discounted profit, where expectation is taken with respect to the risk-neutral pricing measure.

The retailer's discounted expected profit, $R(z_1, z_2, ..., z_N)$, given the suppliers' linear pricing policies $\{\phi_i\}_{i=1}^N$, is:

$$R(z_1, z_2, ..., z_N) = P(z_1, z_2, ..., z_N) - \sum_{i=1}^{N} K_i(\phi_i) z_i. \tag{4}$$

Note that the retailer's discounted expected profit depends on suppliers' policies $\phi_i, i = 1, 2, ..., N$ only through $K_i = K_i(\phi_i), i = 1, 2, ..., N$. Therefore, the retailer responds with the same order quantity to any policy ϕ such that $K_i(\phi) = K$ and the retailer is indifferent between up-front payment ($\alpha = 1$) and on-delivery payment ($\alpha = 0$) provided that w_i^F and w_i^D satisfy the following equation:

$$e^{-r} E[\beta_i] w_i^D = w_i^F. \tag{5}$$

Since suppliers can choose arbitrary values for w_i^F and w_i^D, equation (5) need not hold and the retailer may favor either the up-front or the on-delivery payment policy. This feature can potentially complicate the analysis because the tradeoffs between competition and diversification effects in the supply chain can depend on the payment schemes, even if the policies of payment are restricted to the linear class. Fortunately, Proposition 1 below shows that in *equilibrium*, for all linear policies, the timing of payments does not matter.

Let $S_i(\phi_i, \phi_{-i})$ denote the discounted expected profit of the supplier i given that the other suppliers selects pricing policies ϕ_{-i}. The suppliers are engaged in a Bertrand competition with each other, trying to maximize

$$S_i(\phi_i, \phi_{-i}) = [K_i(\phi_i) - c_i]\, z_i[K_1(\phi_1), K_2(\phi_2), ..., K_N(\phi_N)], \tag{6}$$

where $z_i[K_1(\phi_1), K_2(\phi_2), ..., K_N(\phi_N)]$ is the optimal order quantity placed by the retailer to supplier i, given pricing policies $\phi_i, i = 1, 2, ..., N$. Observe that the supplier i's problem is also a function of $[K_1 = K_1(\phi_1), K_2 = K_2(\phi_2), ..., K_N = K_N(\phi_N)]$ only. Therefore, we can rewrite the suppliers' profit functions as

$$S_i(K_i, K_{-i}) = (K_i - c_i)\, z_i(K_1, K_2, ..., K_N). \tag{7}$$

This observation is helpful in establishing the following proposition:

Proposition 1. *In equilibrium, the retailer, the suppliers and the supply chain are indifferent between methods of payment as long as the policies are in the linear family. In particular, in equilibrium all parties are indifferent between up-front and on-delivery payments.*

Note that if the payment policies were not linear, then the retailer's profit would have been given by:

$$R(z_1, z_2, ..., z_N) = P(z_1, z_2, ..., z_N) - \sum_{i=1}^{N} K_i(\phi_i, z_i) z_i,$$

and the structure of payment policies would have been affected the analysis in complex ways. In what follows, we will assume an up-front payment of K, and interpret it as the wholesale price. However, in light of Proposition 1, we could easily identify an array of equivalent linear policies for which the following analysis will also hold.

4 Ramifications of Risk in the One Supplier Case

To focus on the effects of default risk on supply chains, consider a model with one supplier first. Although a one-supplier model is a simplification, its analysis is not trivial and the insights one gleans from it are valuable. The analysis is based on work by Lariviere and Porteus (2001).

4.1 The Retailer's Problem

With one risky supplier, the retailer's discounted expected revenue, given by equation (3), reduces to $P(z) = e^{-r}sE[\min(D, z\beta)]$. The retailer's expected profit, $R(z)$, given a supplier's wholesale price K, is $R(z) = P(z) - Kz$.

Note that $R(z)$ is concave in z, with $R'(z) = P'(z) - K = e^{-r}sE[\beta\overline{G}(z\beta)] - K$, where $\overline{G}(x) = 1 - G(x)$. The optimal order quantity z satisfies the following first order condition

$$P'(z) \equiv e^{-r}sE[\beta\overline{G}(z\beta)] = K. \tag{8}$$

When $K = c$, the retailer's problem coincides with the problem of a central planner.

Consider two random yields β_1 and β_2. By definition[5], β_2 is stochastically smaller than β_1 (notation: $\beta_2 <_{st} \beta_1$) iff $Pr(\beta_1 > a) \geq Pr(\beta_2 > a)$ for all a. We will equate the notion of increasing credit risk with that of the random yield stochastically decreasing, which will be denoted by $\beta \downarrow_{st}$. Because $\min(D, z\beta)$ is an increasing function of β for every D and z, it follows that the profit of the centralized system $C(z) \equiv e^{-r}sE[\min(D, z\beta)] - cz$ is decreasing as $\beta \downarrow_{st}$. Hence,

Proposition 2. *The optimal profit of the centralized system, $C^* = C(z^*)$, decreases as credit risk increases.*

For the centralized system we would like to characterize the dependence of the optimal order quantity and service level on the level of default risk. Towards this goal define $A(z, \beta) = \beta\overline{G}(z\beta)$. Then, differentiating with respect to β, we obtain $A_\beta(z, \beta) = \overline{G}(z\beta)[1 - h(z\beta)]$, where $h(z) = z\frac{g(z)}{\overline{G}(z)}$ is the generalized failure rate, as defined by Lariviere and Porteus (2001). We would like to identify

[5] For discussion on stochastic order relations, see Shaked and Shanthikumar (1994)

z's for which $A(z,\cdot)$ is increasing. Assume that $h(\cdot)$ is increasing [Increasing Generalized Failure Rate (IGFR) property].[6] Define $\overline{z} = \sup\{z : h(z) \leq 1\}$. Note that $A(\overline{z},\cdot)$ is increasing. Therefore, as $\beta \downarrow_{st}$, $E[A(\overline{z},\beta)]$ decreases. Because $E[A(\overline{z},0)] = 0$ and $\frac{c}{e^{-r}s} > 0$, there exists a random variable β_{max} for which $e^{-r}sE[A(\overline{z},\beta_{max})] < c$ and a solution to equation (8), $z^* \leq \overline{z}$, for $\beta <_{st} \beta_{max}$. The following proposition summarizes the effects of default risk on the optimal order quantity and the service level of the centralized system.

Proposition 3. *Suppose that $G(\cdot)$ is IGFR and for some random variable β_{max}, $e^{-r}sE[A(\overline{z},\beta_{max})] < c$. Then for all $\beta <_{st} \beta_{max}$ as credit risk increases (as $\beta \downarrow_{st}$)*

(i) *The optimal order quantity, $z^{central}$, decreases.*

(ii) *The service level, $Pr(D < z^{central}\beta)$, decreases.*

Proof. See Appendix ■

A stronger assumption on the distribution of the random yield β can make the IGFR requirement unnecessary. For example, if the random yield, β, follows a Bernoulli distribution with the probability of default π then the optimal order quantity for the centralized system is

$$z^{cental} = G^{-1}\left(1 - \frac{c}{e^{-r}(1-\pi)s}\right), \tag{9}$$

and the results of Proposition 3 hold without the IGFR assumption.

4.2 The Supplier's Problem

According to equation (7), the discounted expected profit of the supplier, given that she induces the retailer to order z is given by $S(K) = (K - c)z(K)$. Because there is a one-to-one correspondence between the wholesale price K and the order quantity z, defined by equation (8), we can rewrite the supplier's discounted expected profit as a function of z:

$$S(z) = [P'(z) - c]z. \tag{10}$$

Lemma 1. *There exists a solution to the supplier's problem (10). The optimal order quantity satisfies the following equation:*

$$E\left\{\beta\overline{G}(\beta z^*)[1 - h(\beta z^*)]\right\} = \frac{c}{se^{-r}}. \tag{11}$$

[6] Many common distributions have the IGFR property. For example, any IFR (Increasing Failure Rate) distribution is also IGFR.

Proof. See Appendix ■

In general, equation (11) may have several solutions. To ensure that the supplier's problem is unimodal additional assumptions are needed. Assume that the random yield has a Bernoulli distribution with default probability π. Then the supplier's problem is to maximize

$$S(z) = \left[e^{-r}(1-\pi)s\overline{G}(z) - c\right] z. \tag{12}$$

This problem is equivalent to the problem studied in Lariviere and Porteus (2001) with unit sales revenues given by $s(1-\pi)e^{-r}$ and the next lemma follows directly from their Theorem 1.

Lemma 2. *Suppose that the demand distribution has finite mean, support on $[a, b)$, and function $G(\cdot)$ has an increasing generalized failure rate (IGFR). Then:*

(i) *The first order condition for the supplier's problem is:*

$$\overline{G}(z)\left[1 - h(z)\right] = \frac{c}{s(1-\pi)e^{-r}}. \tag{13}$$

(ii) *The supplier's profit function is unimodal on $[0, +\infty)$, linear and strictly increasing on $[0, a)$, strictly concave on $[a, \overline{z})$, strictly decreasing on $(\overline{z}, +\infty)$. Any solution z^* to equation (13) is unique and must lie in the interval $[a, \overline{z}]$. The supplier's optimal order quantity is either z^* or a.*

Thus, the IGFR property of the demand distribution guarantees the uniqueness of the solution to the supplier's problem.

Next, consider the effects of credit risk. From equation (8), the equilibrium wholesale price is

$$K^* = e^{-r}(1-\pi)s\overline{G}(z^*). \tag{14}$$

Conversely, if the supplier charges a wholesale price, $K^* \geq c$, the retailer orders

$$z^* = G^{-1}\left(1 - \frac{K^*}{e^{-r}(1-\pi)s}\right). \tag{15}$$

Comparing (15) with (9), because $K^* \geq c$, it follows that $z^* \leq z^{central}$.

Similarly to the centralized system, the performance of the decentralized system deteriorates as the default probability increases. Although intuitive, this result is a little less obvious for the decentralized supply chain where the wholesale price, K, is determined as a solution of the Stackelberg game between the supplier and the retailer.

Theorem 1. *For the Stackelberg game between the supplier and the retailer, the equilibrium order quantity, z^*, the optimal supplier's profit, S^*, and the optimal retailer's profit, R^*, are all decreasing in the default probability, π.*

Proof. See Appendix ■

4.3 Supply Chain Echelons and Default Risk Burden

Default risk is detrimental for a supply chain. However, Theorem 1 does not specify which firm incurs the brunt of the losses as the supplier's default risk increases. To answer this question, define $\eta(\pi) \equiv \frac{S^*}{R^*}$ as a ratio of equilibrium profits of the supplier and the retailer. If $K^*(\pi)$ is the equilibrium wholesale price and $z^*(\pi)$ is the equilibrium order quantity corresponding to default probability π (see (14) and (13)) , then using definitions of R^* and S^* we can obtain a lower bound for $\eta(\pi)$ (the explicit dependence on π will generally be omitted)

$$\eta(\pi) = \frac{S^*}{R^*} = \frac{(K^* - c)z^*}{e^{-r}(1-\pi)sE\min(D, z^*) - K^*z^*} \geq \frac{K^* - c}{e^{-r}(1-\pi)s - K^*} = z^*\frac{g(z^*)}{G(z^*)} = \gamma[z^*(\pi)],$$

where $\gamma(z) = z\frac{g(z)}{G(z)}$. The lower bound $\gamma[z^*(\pi)]$ represents the ratio of supplier's and retailer's profit per each sold unit. Suppose that for some $\pi_{low} < \pi_{high}$, and the corresponding the optimal quantities $[z^*(\pi_{high}), z^*(\pi_{low})]$, function $\gamma(\cdot)$ is increasing (decreasing). Then as the default probability $\pi \in [\pi_{low}, \pi_{high}]$ increases, the ratio

$$\frac{K^* - c}{e^{-r}(1-\pi)s - K^*} = \gamma[z^*(\pi)]$$

decreases (increases), that is the marginal profit of the supplier is diminishing faster (slower) than the marginal profit of the retailer.

While the lower bound, $\gamma[z^*(\pi)]$, is not the same as the ratio of profits $\eta(\pi) = \frac{S^*}{R^*}$, it is easier to analyze and it serves as an approximation, which would be fairly precise if the probability that the retailer sells the entire order z^* is high.

This probability is related to the service level of the system, defined as $Pr(D < z^*\beta)$. This is another important measure of the supply chain performance and is equal to (using (14))

$$Pr(D < z^*\beta) = (1-\pi)G(z^*) = \frac{e^{-r}(1-\pi)s - K^*}{e^{-r}s}. \tag{16}$$

Observe that the ratio of the service level of the decentralized system, $\frac{e^{-r}(1-\pi)s - K^*}{e^{-r}s}$, and the service level of the centralized system, $\frac{e^{-r}(1-\pi)s - c}{e^{-r}s}$, depends on function $\gamma(\cdot)$:

$$\frac{e^{-r}(1-\pi)s - K^*}{e^{-r}(1-\pi)s - c} = \frac{1}{1 + \gamma[z^*(\pi)]} \tag{17}$$

Again, suppose that for some $\pi_{low} < \pi_{high}$, and the corresponding the optimal quantities $z^*(\pi_{high})$ and $z^*(\pi_{low})$, function $\gamma(\cdot)$ is increasing (decreasing). Then as $\pi \in [\pi_{low}, \pi_{high}]$ increases, the ratio in (17) increases (decreases).

The ratio of service levels is equal to the conditional probability of meeting customer demand in the decentralized system, given that the demand is met in the centralized system. When $\gamma(\cdot)$ is decreasing, we overestimate this conditional probability if we ignore the credit risk in the system. Thus, we underestimate the severity of the drop in the service level. On the other hand, when $\gamma(\cdot)$ is increasing, by ignoring credit risk, we are being too pessimistic about the service levels in the decentralized system.

Numerical results suggest that the actual ratio of optimal supplier and retailer's profits, $\eta(\pi)$, behaves similarly to the lower bound, $\gamma[z^*(\pi)]$, as illustrated by the following example.

Example 1.
Assume: $s = 100, c = 30$ and $r = 0.1$. This example presents results for the cases of exponential demand with mean 150 and normal demand with mean 150 and standard deviation 60. According to Theorem 1, as credit risk increases, the optimal supplier's profit, the optimal retailer's profit and the coordinated channel profit are decreasing. These properties are illustrated in Panel A of Figure 1.

Panel B of Figure 1 shows that the ratio $\frac{S^*}{R^*}$ and its lower bound $\gamma[z^*(\pi)]$ could be increasing or decreasing, depending on the conditions on the demand distribution discussed earlier.

Panel C of Figure 1 demonstrates the behavior of another measure of the supply chain performance, the ratio of profits of the decentralized system $(S^* + R^*)$ and the centralized system (C^*). The figure shows that as the default risk increases, the ratio $\frac{S^*+R^*}{C^*}$ is slightly increasing in π. For the normal demand the ratio $\frac{S^*+R^*}{C^*}$ is decreasing for large π. ■

5 The Effect of Correlation

As was shown in section 4, default risk reduces firms' profits as well as the channel profit. To moderate default risk exposure, the retailer might consider placing orders with several suppliers. Ceteris paribus, if the wholesale prices are exogenously fixed, because of diversification, the retailer benefits from decreasing correlation between suppliers defaults. This section shows how the correlation affects retailer and supplier profits if wholesale prices are determined endogenously.

In this section we will assume that the random yields, β_i, of the suppliers follow Bernoulli distributions with probabilities of default $\pi_i, i = 1, 2, ..., N$. Let $d_i \in \{0, 1\}$ be the number of defaults of supplier i during the production period. The joint default probabilities of the suppliers, $p_{d_1 d_2 ... d_N} = \Pr[\beta_i = 1 - d_i, i = 1, ..., N]$ are known to the retailer and the suppliers.

Note that the correlation effect is especially profound when the probability of a default over the fixed time horizon is small. To see this, let $N = 2$ and let ρ be the default correlation over a finite horizon. Then:

$$\rho = \frac{p_{11} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

For $\pi_1 = \pi_2 = \pi$, and π small, we have:

$$p_{11} = \rho\pi + (1-\rho)\pi^2 \approx \rho\pi.$$

Further, given supplier 1 has defaulted, the probability that supplier 2 defaults is given by

$$\pi_{2|1} = \frac{p_{11}}{\pi_1} \approx \rho.$$

These results show that when events are rare, the default probability dependence is largely determined by the correlation coefficient.

There are a number of approaches to modeling the effects of increasing codependence between supplier defaults on the joint default distribution, $p_{d_1 d_2 \ldots d_N}$. While Pearson's linear correlation coefficient works well as a codependence measure for models with elliptical distributions, Embrechts, McNeil and Straumann (2002) show that it might be inadequate for non-elliptic problems. Copula functions have been proposed as a modeling alternative to linear correlation by a number of authors (see, Nelsen (1999) and Embrechts, Lindskog and McNeil (2003)). However, the copula methodology has been developed for continuous distributions and the choice of the appropriate copula class is a non-trivial task. For the purposes of this paper, it is sufficient to model changes in the joint default distributions, $p_{d_1 \ldots d_N}$, directly. Specifically, let $N = 2$. The joint default distribution and the marginal probabilities satisfy the following:

$$p_{00}, p_{01}, p_{10}, p_{11} \geq 0; \quad p_{00} + p_{01} + p_{10} + p_{11} = 1;$$
$$p_{00} + p_{01} = 1 - \pi_1; \quad p_{00} + p_{10} = 1 - \pi_2; \quad p_{11} + p_{01} = \pi_2; \quad p_{11} + p_{10} = \pi_1.$$

If the defaults are perfectly positively correlated, then $p_{01} = p_{10} = 0$ and $p_{00} = 1 - \pi_1 = 1 - \pi_2$ (hence $\pi_1 = \pi_2$). As the correlation decreases, p_{01} and p_{10} increase and p_{00} decreases. When defaults are perfectly negatively correlated, $p_{11} = p_{00} = 0$ and $p_{01} = 1 - \pi_1, p_{10} = 1 - \pi_2$.

The analysis will be performed for the model with two suppliers. However, results could be extended to the model with three or more suppliers as discussed at the end of the section.

5.1 Deterministic Demand

With one supplier the problem was not trivial to solve. With two or more suppliers, the analysis becomes even more complex because one needs to find an equilibrium solution to the game among suppliers. To simplify the analysis we first consider the case when demand is deterministic.

5.1.1 The Retailer's Problem

Given information about the default distribution and the supplier's wholesale prices $K_i, i = 1, 2$, the retailer determines how much to order from each of the suppliers so as to maximize

$$R(z_1, z_2) = P(z_1, z_2) - K_1 z_1 - K_2 z_2, \tag{18}$$

where

$$P(z_1, z_2) = e^{-r} s \left[p_{01} \min(D, z_1) + p_{10} \min(D, z_2) + p_{00} \min(D, z_1 + z_2) \right]. \tag{19}$$

The solution to the retailer's problem is described in the following proposition.

Proposition 4. *Assume that $e^{-r}s(1 - \pi_i) \geq K_i, i = 1, 2$. Then*

$$(z_1^*, z_2^*) = \begin{cases} (D, D) & \text{if } K_1 \leq e^{-r}sp_{01} \text{ and } K_2 \leq e^{-r}sp_{10} \\ (0, D) & \text{if } e^{-r}sp_{01} < K_1 < e^{-r}s(1 - \pi_1) \text{ and } K_2 < K_1 + e^{-r}s(\pi_1 - \pi_2) \\ (D, 0) & \text{if } e^{-r}sp_{10} < K_2 < e^{-r}s(1 - \pi_2) \text{ and } K_2 > K_1 + e^{-r}s(\pi_1 - \pi_2) \\ z_1^* + z_2^* = D; z_i^* \geq 0 & \text{if } K_1 > e^{-r}sp_{01},\ K_2 > e^{-r}sp_{10}, \text{ and } K_2 = K_1 + e^{-r}s(\pi_1 - \pi_2) \end{cases}$$

Proof. See Appendix ■

Figure 2 provides a graphical representation of the retailer's response described in Proposition 4.

As the correlation between supplier defaults increases, the probabilities that only one supplier delivers the order, p_{01} and p_{10}, decrease, and the probability that both suppliers deliver orders, p_{00}, increases. Therefore, the region where the retailer order from both suppliers shrinks. Consequently, the optimal order quantities to each supplier are nonincreasing in the default correlation.

5.1.2 Equilibrium Solution of the Game between Suppliers

The suppliers compete by selecting wholesale prices K_i that maximize their discounted expected profits as given in equation (7). Based on the retailer's response function the solution to the game between the suppliers is given in the following proposition.

Proposition 5. *The equilibrium solution to the game between suppliers is unique and*

(i) *If $e^{-r}sp_{01} > c_1$ and $e^{-r}sp_{10} > c_2$, then $(K_1^*, K_2^*) = (e^{-r}sp_{01}, e^{-r}sp_{10})$. The retailer's order quantities are (D, D).*

(ii) *If* $e^{-r}sp_{01} > c_1$ *and* $e^{-r}sp_{10} \leq c_2$*, then* $(K_1^*, K_2^*) = (c_2 - e^{-r}s(\pi_1 - \pi_2), c_2)$*. The retailer's order quantities are* $(D, 0)$*.*

(iii) *If* $e^{-r}sp_{01} \leq c_1$ *and* $e^{-r}sp_{10} > c_2$*, then* $(K_1^*, K_2^*) = (c_1, c_1 + e^{-r}s(\pi_1 - \pi_2))$*. The retailer's order quantities are* $(0, D)$*.*

Figure 2 shows the unique equilibrium solution described in Proposition 5.

As the correlation between supplier defaults increases, the equilibrium wholesale prices decrease. To understand the intuition behind this result one can think of the default as being one of the attributes of the product offered by the suppliers. When supplier defaults are perfectly correlated, the goods the suppliers sell are perfect substitutes and the force of competition between suppliers drives the wholesale prices down to the production costs (this is an example of Bertrand competition). As the defaults become less correlated, the goods offered by suppliers become less substitutable, and the competition is less effective in holding the prices down. Ultimately, when supplier defaults are perfectly negatively correlated, the suppliers deliver goods in different probabilistic states of nature and do not compete.

5.1.3 Defaults Correlation and Supply Chain Profits

Part (i) of Proposition 5 is the most relevant for the study of correlation effects, because in this case both suppliers participate in the game. Using expressions for the equilibrium prices and order quantities, under assumptions of part (i) in Proposition 5, the equilibrium retailer's, suppliers', system's, and coordinated system's profits are

$$R^* = e^{-r}sD\left(p_{01} + p_{10} + p_{00}\right) - e^{-r}sp_{01}D - e^{-r}sp_{10}D \quad = e^{-r}sp_{00}D, \tag{20a}$$

$$S_1^* = \left(e^{-r}sp_{01} - c_1\right)D, \tag{20b}$$

$$S_2^* = \left(e^{-r}sp_{10} - c_2\right)D, \tag{20c}$$

$$U^* = \left(e^{-r}sp_{01} - c_1\right)D + \left(e^{-r}sp_{10} - c_2\right)D + e^{-r}sp_{00}D \quad = e^{-r}s(1 - p_{11})D - c_1D - c_2D, \tag{20d}$$

$$C^* = e^{-r}sD\left(p_{01} + p_{10} + p_{00}\right) - c_1D - c_2D \quad = e^{-r}s(1 - p_{11})D - c_1D - c_2D. \tag{20e}$$

Using these explicit expressions for profits we obtain the following result:

Theorem 2. *If* $e^{-r}sp_{01} > c_1$ *and* $e^{-r}sp_{10} > c_2$*, then the channel profit is equal to the coordinated channel profit (*$U^* = C^*$*) and, as the correlation between supplier defaults increases:*

(i) *The supply chain profit,* $U^* = C^*$*, decreases*

(ii) *The retailer's profit,* R^**, increases*

(iii) *The suppliers' profits,* S_1^* *and* S_2^**, decrease.*

All things being equal, the retailer would prefer that the suppliers have highly positively correlated default processes. Positive correlation between defaults leads to lower wholesale prices, compensating the retailer for the loss of diversification benefits. Conversely, all things being equal, each supplier would prefer that their competitor have a default process that is highly negatively correlated with their own default processes. When defaults are perfectly negatively correlated there is no competition between the suppliers (in the probabilistic states of nature where one of the suppliers survived the other one defaulted), and each supplier behaves as a monopolist, extracting all of the system profits.

If it were feasible, the suppliers (and the channel) would benefit by decreasing their default correlation. The correlation between defaults can be reduced by using different production technologies, different raw materials sources, by placing production facilities in different parts of the country (or different countries). This might provide firms with incentives to expand their global operations.

Finally, note that the supply chain profit increases as the correlation of defaults decreases. Therefore, what is good for the supplier is also good for the channel, but detrimental for the retailer.

5.2 Stochastic Demand

We will now extend the analysis to the case where demand is stochastic and absolutely continuous.

5.2.1 The Retailer's Problem

Given wholesale prices K_1 and K_2, the retailer maximizes her discounted expected profit,

$$R(z_1, z_2) = P(z_1, z_2) - K_1 z_1 - K_2 z_2, \tag{21}$$

where

$$\begin{aligned} P(z_1, z_2) =& e^{-r} s E\left[\min(D, z_1\beta_1 + z_2\beta_2)\right] = \\ =& e^{-r} s \left\{p_{01} E\left[\min(D, z_1)\right] + p_{10} E\left[\min(D, z_2)\right] + p_{00} E\left[\min(D, z_1 + z_2)\right]\right\}. \end{aligned} \tag{22}$$

The following proposition summarizes the solution of the retailer's problem with stochastic demand.

Proposition 6. *The optimal order quantities, (z_1, z_2), for the problem in (21), (22) satisfy the following systems of equations:*

$$\text{If} \begin{cases} K_2 \geq \frac{p_{00}}{1-\pi_1} K_1 + e^{-r} s p_{01}, \\ K_1 \leq e^{-r} s (1 - \pi_1) \end{cases} \qquad \text{then} \begin{cases} e^{-r} s (1 - \pi_1) \overline{G}(z_1) = K_1 \\ z_2 = 0. \end{cases} \tag{23a}$$

$$\text{If } \begin{cases} K_1 \geq \frac{p_{00}}{1-\pi_2} K_2 + e^{-r} s p_{10}, \\ K_2 \leq e^{-r} s (1-\pi_2) \end{cases} \quad \text{then } \begin{cases} z_1 = 0 \\ e^{-r} s (1-\pi_2) \overline{G}(z_2) = K_2. \end{cases} \tag{23b}$$

$$\text{If } \begin{cases} K_1 < \frac{p_{00}}{1-\pi_2} K_2 + e^{-r} s p_{10}, \\ K_2 < \frac{p_{00}}{1-\pi_1} K_1 + e^{-r} s p_{01} \end{cases} \quad \text{then } \begin{cases} e^{-r} s \left[p_{01} \overline{G}(z_1) + p_{00} \overline{G}(z_1 + z_2) \right] = K_1 \\ e^{-r} s \left[p_{10} \overline{G}(z_2) + p_{00} \overline{G}(z_1 + z_2) \right] = K_2. \end{cases} \tag{23c}$$

$$\text{Otherwise} \quad \begin{cases} z_1 = 0 \\ z_2 = 0. \end{cases} \tag{23d}$$

Proof. See Appendix ■

Figure 3 provides a graphical representation of the retailer's response function described in Proposition 6.

Note that the problem where the retailer controls wholesale prices could be readily solved using results presented here. The retailer's optimal policy would be to lower wholesale prices to production costs ($K_i = c_i$) and then order according to Proposition 6 (or Proposition 4 if demand, D, is deterministic). Unfortunately, the problem with the suppliers determining wholesale prices, which is the main focus of this paper, is significantly more complicated.

Define z_k^{mon} to be equilibrium solution of the model where supplier k is a monopolist (equation (13)). The following result will be needed in the subsequent analysis to prove the existence of an equilibrium.

Corollary 1. *Suppose that $p_{10} > 0$ and $p_{01} > 0$. If for all $(z_1, z_2) \in [0, z_1^{mon}] \times [0, z_2^{mon}]$*

$$p_{01} p_{10} g(z_1) g(z_2) + p_{00} g(z_1 + z_2) \left[p_{01} g(z_1) + p_{10} g(z_2) \right] > 0. \tag{24}$$

Then, for any supplier i, the optimal order quantity $z_i(K_i, K_{-i})$ is a continuous function of K_i for a fixed wholesale price of the other supplier K_{-i}.

Proof. See Appendix ■

For the rest of the paper we will assume that condition (24) is satisfied.

5.2.2 Equilibrium Solution of the Suppliers' Game

The suppliers maximize their discounted expected profits given by equation (7). Observe that $K_i > e^{-r}(1-\pi_i)s, i = 1, 2$ is a dominated strategy for each of the suppliers. Therefore, it is sufficient to consider suppliers pricing policies restricted in the rectangle $[0, e^{-r}(1-\pi_1)s] \times [0, e^{-r}(1-\pi_2)s]$. By Corollary 1, $z(\cdot,\cdot)$ is a continuous function. Therefore, from Theorem 1.1 in Glicksberg (1952) we derive the following:

Proposition 7. *There exists a mixed-strategy equilibrium solution to the suppliers' game.*

It is difficult to show, however, that there exists a pure-strategy equilibrium for this game. The game is not supermodular, therefore, the results in Topkis (1998) cannot be applied. It is also difficult to produce parsimonious conditions that would ensure quasi-concavity of the suppliers' profit functions, even though we can verify this property for particular distributions (normal, exponential). Therefore, we cannot invoke results from Debreu (1952). For simplicity, assume that the problem is symmetric, that is $c_1 = c_2 = c$ and $\pi_1 = \pi_2 = \pi$ (consequently, $p_{01} = p_{10}$). Then, if there exists a symmetric pure-strategy equilibrium, it can be characterized in the next proposition.

Proposition 8. *If there exists a symmetric pure-strategy equilibrium, then the equilibrium order quantities, $z_1^* = z_2^* = z^*$, satisfy*

$$p_{01}\overline{G}(z)[1-h(z)] + p_{00}\overline{G}(2z)\left[1-\frac{1}{2}h(2z)\right] + \frac{p_{00}^2 g^2(2z)z}{p_{10}g(z)+p_{00}g(2z)} = \frac{c}{e^{-r}s}. \tag{25}$$

The equilibrium wholesale prices are

$$K_1^* = K_2^* = e^{-r}s\left[p_{01}\overline{G}(z^*) + p_{00}\overline{G}(2z^*)\right]. \tag{26}$$

Proof. See Appendix ∎

Based on Proposition 8, a symmetric pure-strategy equilibrium can be computed, if it exists, by first solving equation (25) and then computing the corresponding equilibrium wholesale price, K^*, using equation (26).

5.2.3 Default Correlation and Supply Chain Profits

While for arbitrary correlations it is difficult to characterize the equilibrium solution, for the special cases of perfect positive and perfect negative correlation between defaults the analysis is tractable. For example, assume that the suppliers' default events are perfectly positively correlated ($p_{01} = p_{10} = 0$, $p_{00} = 1-\pi$). In this case, the wedge-shaped region of the shared retailer's business in Figure 3 shrinks to a line $K_2 = K_1$ and the supplier who charges a lower price is awarded all of the retailer's business. The suppliers' game turns into a classical Bertrand competition, where the winner is the supplier with the lowest production cost. When the defaults are perfectly negatively correlated ($p_{00} = p_{11} = 0$, $p_{01} = 1-\pi_1$, $p_{10} = 1-\pi_2$), the wedge-shaped region of the shared retailer's business stretches to fill the entire rectangle $[0, e^{-r}s(1-\pi_1)] \times [0, e^{-r}s(1-\pi_2)]$. The optimal order quantity satisfies (23c). However, because $p_{00} = 0$, (23c) becomes separable with optimal order quantity, z_i, depending only on the value of K_i. Therefore, each supplier solves a single supplier problem (12), selects the monopolist order quantity z_i^{mon} that satisfies equation (13), and charges the monopolist's wholesale price. To study the supply chain performance at intermediate values of the default correlation, we resort to numerical analysis.

Example 2. (Arbitrary Correlation. Exponential Demand Distribution)
Suppose that the demand distribution is exponential with mean 150 units and that the values of the other parameters are $s = 100, c_1 = c_2 = c = 10, r = 0.1, \pi_1 = \pi_2 = \frac{1}{2}$.[7] Using the two-step procedure described in the previous subsection we establish the symmetric equilibrium order quantity, z^*, and wholesale price, K^*, for different values of p_{00}.

Figure 4 illustrates that as the correlation between suppliers' defaults increases, the system profit and suppliers' profits decrease, while the retailer's profit increases. ■

Similar results are obtained for other demand distributions. Just as in the case of deterministic demand, we observe that a positive correlation between the defaults induces more intense competition between the suppliers, benefiting the retailer. While the supply chain as a whole benefits from diversification, the retailer makes the least profits when the defaults are perfectly negatively correlated. Because of this conflict of interests, the responsibilities of the central planner in a supply chain cannot be delegated to the retailer.

5.3 Multiple suppliers

The insights obtained using the two-supplier model can be extended to the model with three or more suppliers. Retaining the assumption of the Bernoulli yields, $\{\beta_i\}_{i=1}^N$, for suppliers, we can write the retailer's problem as follows:

$$\max_{z_1 \geq 0, z_2 \geq 0, \ldots, z_N \geq 0} \left\{ e^{-r} s \sum_{d_1=0}^{1} \sum_{d_2=0}^{1} \ldots \sum_{d_N=0}^{1} E\left[\min\left(D, \sum_{i=1}^{N} (1-d_i) z_i \right) \right] p_{d_1 d_2 \ldots d_N} - \sum_{i=1}^{N} K_i z_i \right\}, \quad (27)$$

where d_i is the number of defaults of supplier i and $p_{d_1 d_2 \ldots d_N} = \Pr\left[\beta_1 = 1 - d_1, \ldots, \beta_N = 1 - d_N\right]$ is the joint default distribution of the N suppliers. Problem (27) is concave and the first order conditions are given by the following equations (for $j = 1, \ldots, N$ and assuming nonzero order to all

[7]Note that a value $\pi = \frac{1}{2}$ is extremely high from a practical perspective, however, this is the only value that allows us to consider the full range of correlations (from perfect negative to perfect positive) in a symmetric game.

suppliers)

$$\sum_{d_1=0}^{1} \cdots \sum_{d_{j-1}=0}^{1} \sum_{d_{j+1}=0}^{1} \cdots \sum_{d_N=0}^{1} \overline{G}\left[\sum_{i\neq j}(1-d_i)z_i + z_j\right] p_{d_1\ldots d_{j-1}0\, d_{j+1}\ldots d_N} = \frac{K_j}{e^{-r}s}. \tag{28}$$

The solution of the retailer's problem provides the optimal order quantities to each of the suppliers, $\{z_j(K_1,...,K_N)\}_{j=1}^N$, given the values for the wholesale prices, $\{K_j\}_{j=1}^N$. Supplier j's problem is

$$\max_{K_j\geq 0}(K_j - c_j)z_j(K_1,...,K_N) \tag{29}$$

As is the two-supplier model, finding a non-cooperative equilibrium is difficult, in general. However, if we assume that the demand, D, is deterministic, then, similar to the two-supplier model, the equilibrium wholesale prices turn out to be $(K_1^*, K_2^*, ..., K_N^*) = (e^{-r}s\, p_{01\ldots 1}, e^{-r}s\, p_{10\ldots 1}, ..., e^{-r}s\, p_{11\ldots 0})$, provided that $K_j^* > c_j$ for all $j = 1,2,...,N$. The equilibrium order quantities are $(z_1^*, z_2^*, ..., z_N^*) = (D, D, ..., D)$.

Consider $p_{1\ldots 0\ldots 1} = \Pr[\beta_j = 1, \beta_i = 0 \text{ for } i \neq j]$. As the codependence between defaults of suppliers k and j increases,

$$\Pr[\beta_j = 1, \beta_i = 0 \text{ for } i \neq j] = \Pr[\beta_j = 1, \beta_k = 0] \cdot \Pr[\beta_i = 0, \text{ for } i \neq k \;\&\; i \neq j \mid \beta_j = 1, \beta_k = 0]$$

and $\Pr[\beta_j = 1, \beta_k = 0]$ decreases which[8] implies that the prices K_i and K_j are decreasing. Therefore, the suppliers have an incentive to reduce their default correlation. Similarly, one can show that prices of other suppliers are increasing. Observe that as the number of suppliers, N, increases, the equilibrium wholesale prices tend to decrease. It is easiest to show this intuitive result when defaults are independent and all marginal default probabilities are equal to 0.5. In general, the observation follows from the fact that

$$\begin{aligned}\Pr[\beta_1 = 0, ..., \beta_j = 1, ..., \beta_N = 0] = &\Pr[\beta_1 = 0, ..., \beta_j = 1, ..., \beta_N = 0, \beta_{N+1} = 0] + \\ &\Pr[\beta_1 = 0, ..., \beta_j = 1, ..., \beta_N = 0, \beta_{N+1} = 1].\end{aligned} \tag{30}$$

The equilibrium retailer's (R^*), suppliers' (S_j^*), system's (U^*), centralized system's (C^*) profits are

$$R^* = e^{-r}sD\left(1 - p_{11\ldots 1} - \sum_{i=1}^{N} p_{1\ldots 101\ldots 1}\right), \tag{31a}$$

$$S_j^* = \left(e^{-r}sp_{1\ldots 101\ldots 1} - c_j\right)D, \quad j = 1,...,N, \tag{31b}$$

$$U^* = C^* = e^{-r}sD\left(1 - p_{11\ldots 1} - \sum_{i=1}^{N} c_i\right). \tag{31c}$$

[8]assuming that $\Pr[\beta_i = 0, \text{ for } i \neq k \;\&\; i \neq j \mid \beta_j = 1, \beta_k = 0]$ does not change

If the increasing codependence between defaults translates into increasing $p_{11\ldots1}$, then the equilibrium system profits, $U^* = C^*$, are decreasing. As the codependence between defaults of suppliers k and j increases, the equilibrium profits of these suppliers are decreasing and the equilibrium profits of their competitors are increasing. Let's consider increasing correlation between defaults of suppliers 1 and 2. The retailer's profit could be rewritten as

$$\begin{aligned}R^* =& e^{-r}sD\Bigg\{1 - p_{11}\Bigg(\Pr[\beta_i = 0, i = 3, ..., N \mid \beta_1 = 0, \beta_2 = 0] + \\ & \sum_{j=3}^{N} \Pr[\beta_j = 1, \beta_i = 0, i = 3, ..., N \ \&\ i \neq j \mid \beta_1 = 0, \beta_2 = 0] - \\ & \Pr[\beta_i = 0, i = 3, ..., N \mid \beta_1 = 0, \beta_2 = 1] - \Pr[\beta_i = 0, i = 3, ..., N \mid \beta_1 = 1, \beta_2 = 0]\Bigg) - \\ & \pi_1 \Pr[\beta_i = 0, i = 3, ..., N \mid \beta_1 = 0, \beta_2 = 1] - \pi_2 \Pr[\beta_i = 0, i = 3, ..., N \mid \beta_1 = 1, \beta_2 = 0]\Bigg\}\end{aligned} \tag{32}$$

where $p_{11} = \Pr[\beta_1 = 0, \beta_2 = 0]$. The retailer's profit is increasing in default correlation if the expression next to p_{11} is negative and is decreasing in default correlation if it is negative. Consider, for example, special cases with $N = 2, 3, 4$. Assume that the defaults of the suppliers 1 and 2 are independent from defaults of the suppliers with higher numbers. For $N = 2$ we recover expression (20a) and conclude that the retailer's profit is increasing in the default correlation. For $N = 3$,

$$R^* = e^{-r}sD\left[1 - p_{11}\left(1 - 2\pi_3\right) - (\pi_1 + \pi_2)\pi_3\right] \tag{33}$$

Therefore, if the marginal default probability of the supplier 3, $\pi_3 < 0.5$, then the retailer's profit is decreasing in correlation between defaults of suppliers 1 and 2. Conversely, if $\pi_3 > 0.5$ then the retailer's profit is increasing in correlation between defaults of suppliers 1 and 2. Suppose, $N = 4$, then

$$\begin{aligned}R^* = e^{-r}sD\big\{&1 - p_{11}\big(\Pr[\beta_3 = 1, \beta_4 = 0] + \Pr[\beta_3 = 0, \beta_4 = 1] - \Pr[\beta_3 = 0, \beta_4 = 0]\big) - \\ &(\pi_1 + \pi_2)\Pr[\beta_3 = 0, \beta_4 = 0]\big\}\end{aligned} \tag{34}$$

If $\Pr[\beta_3 = 1, \beta_4 = 0] + \Pr[\beta_3 = 0, \beta_4 = 1] - \Pr[\beta_3 = 0, \beta_4 = 0] > 0$, then the retailer's equilibrium profit is decreasing in default correlation of suppliers 1 and 2. Otherwise, the profit is increasing in the default correlation.

Thus, for $N > 2$ the retailer no longer automatically benefits from the increasing default correlation. Instead, we need to verify condition (32).

6 Conclusion

The recent experience of high levels of corporate defaults, together with awareness of possibilities of supply disruption, have reinforced the importance of credit risk management, not only as a treasury function, but also in the context of operational planning, as well as highlighted the need for backup agreements, contingency measures, and alternative sources of supplies. A growing literature has emerged that address many of the consequences of supply chain risk. We contribute to this literature by focusing on the impact of supplier competition in a market where the retailer is considering diversification as a strategy to reduce supply chain risk. We believe that this paper is one of the first to address supply-chain management questions in a model where competition between suppliers implicitly affect equilibrium wholesale prices, in a way that depends on the degree of default correlation.

Using a simple one-period model of a supply chain with one retailer and multiple risky suppliers, this paper studies questions of supplier selection, pricing and ordering policies among firms. In our model, the suppliers compete for business from the retailer, and are, collectively, Stackelberg leaders in a game with the retailer.

Although, in general, the timing of the payments from the retailer to the suppliers may be important, we identified a family of general linear pricing policies, such that, in equilibrium, the suppliers, the retailer, and the channel are not concerned with the timing of payments. A positive side-effect of this important result is a tremendous reduction in the complexity of the subsequent analysis.

Not surprisingly, default risk has detrimental effect on firms in a supply chain. For the one-supplier model, we identify conditions on the demand distribution function that show where in the supply chain the brunt of supply chain risk is borne. In particular, the rate of profits decline for firms in different echelons of the supply chain depends on the concavity or convexity of the demand's cumulative distribution function.

With more than one supplier, the retailer may decide to hedge default risk by splitting orders. If the wholesale prices were *exogenously* fixed, then, as one would expect, the negative correlation between default events yields higher diversification benefits to the retailer. However, in our competitive environment, the wholesale prices are determined *endogenously* by the suppliers. We are able to find equilibrium solutions analytically when demand is deterministic or when demand is stochastic and default correlation is either one or minus one. For the model with stochastic demand and arbitrary correlation we compute the equilibrium solution numerically. The analysis of the equilibrium solution shows that the positive correlation between default events stimulates competition between suppliers leading to lower wholesale prices. The benefits to the retailer, due to the lower wholesale prices, far outweigh the losses due to the weaker diversification. Therefore, contrary to initial intuition about the advantages of diversification, positive default correlation benefits the retailer. We also show that a negative default correlation benefits the suppliers and the channel as a whole. Thus, incentives of the retailer and the channel are misaligned. The retailer should not be delegated to coordinate the channel. Further, once the suppliers are chosen, any actions they can take to reduce their correlation will be advantageous for them. For example, they may attempt to sell to different customers, use different production technologies, procure from different raw materials sources, and reduce exposures to common country specific risks or common catastrophic events.

In our analysis we have made several simplifying assumptions. For example, we assumed that the default and demand processes are independent, the default distribution of suppliers does not depend on the order quantities, the production lead times for both suppliers are equal. However, even in our simple case, including supply risk considerations into operational planning significantly affects ordering and pricing decisions in a supply chain and alters the nature of competition among firms. It remains for future research to study the effects of weakening these assumptions.

Rather than use multiple suppliers to hedge default risk of a supplier, a retailer could enter into a financial contract with an investment bank, whereby the retailer would be compensated if the supplier, or any one of a set of suppliers defaulted. While a mushrooming market for these insurance products (in particular, credit default swaps and first to default contracts) has emerged, these contracts typically are linked to a firms financial health, and are triggered by events such as a downgrade in bond ratings, a missed coupon, or a formal declaration of entering Chapter 11. Very seldom are contracts designed that are linked to performance of specific supply contracts. Indeed, even when limited to financial conditions, asymmetric information, with the resulting moral hazard implications, is of a great concern to investment banks who might be less informed about the condition of a firm than the retailer who might have privileged relationships with some of the suppliers. Since the typical response to asymmetric information is for a bank to increase the cost of default insurance above its "true" value, the retailer might be better off investigating operational solutions, such as splitting orders, rather than financial solutions. It remains for future research is consider contracting arrangements that possibly combine financial hedging with operational planning tools.

Figure 1: Comparisons of profits for normal and exponential demand distributions

The figure shows the optimal profits, ratio of supplier and retailer profits with lower bounds, and ratio of decentralized to centralized system profits for the normal and exponential demands. The case parameters are s=100; c=30; r=0.10. For the normal demand, the mean is 150 and std. deviation is 60. For the exponential demand, the mean is 150 units.

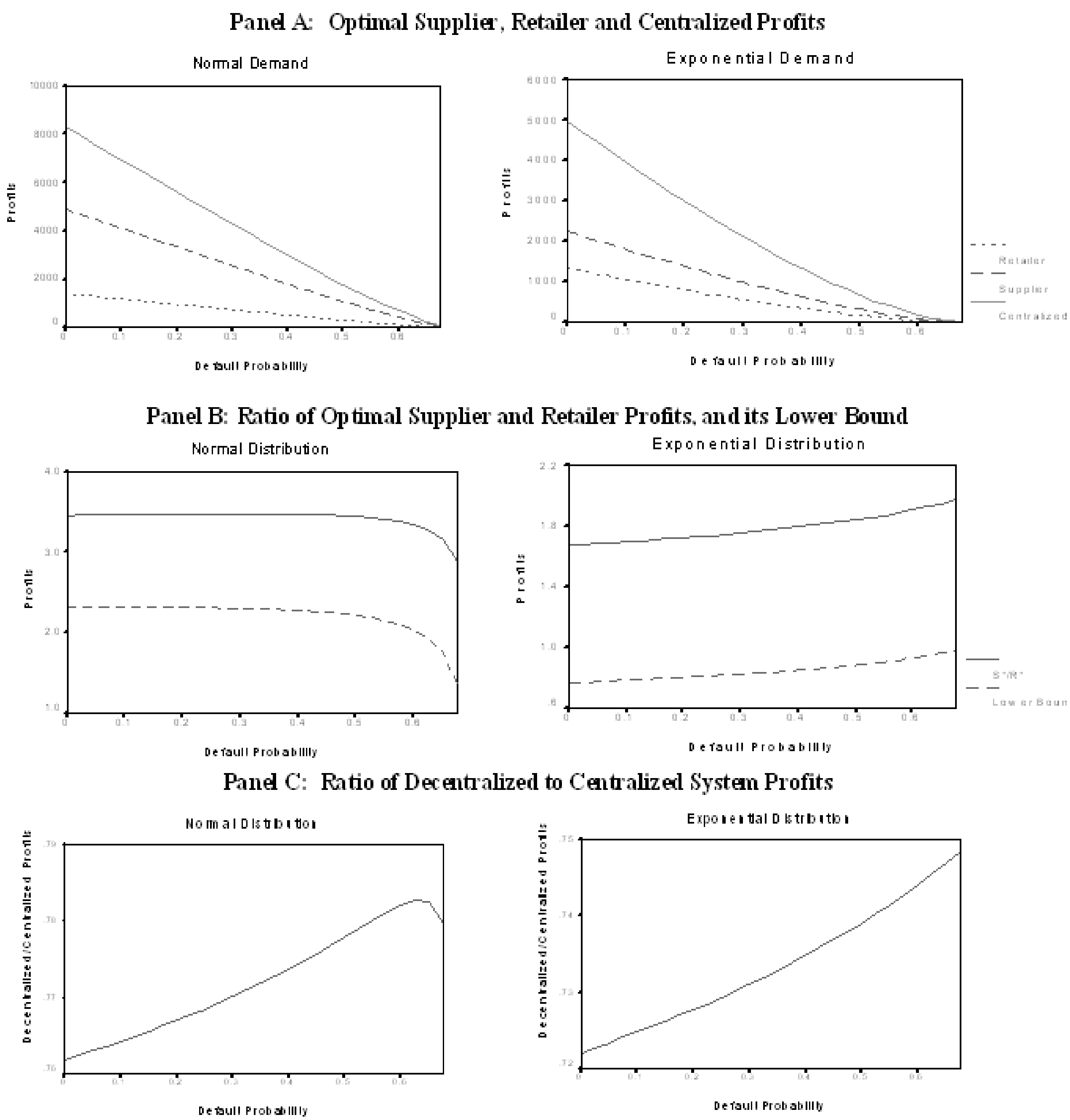


Figure 2: Retailer's response function and equilibrium solution to the game between suppliers when demand is deterministic.

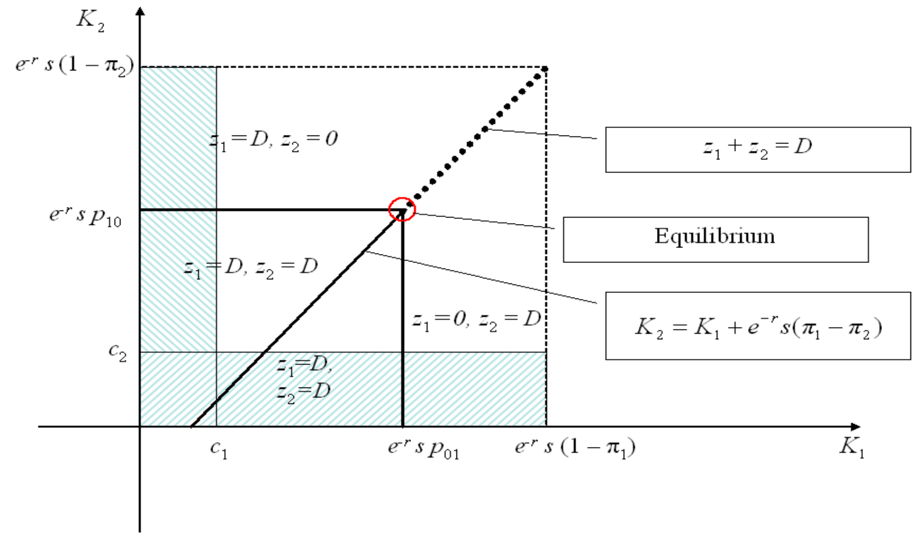


Figure 3: Retailer's response function to wholesale prices $K_i, i = 1, 2$ when demand is stochastic.

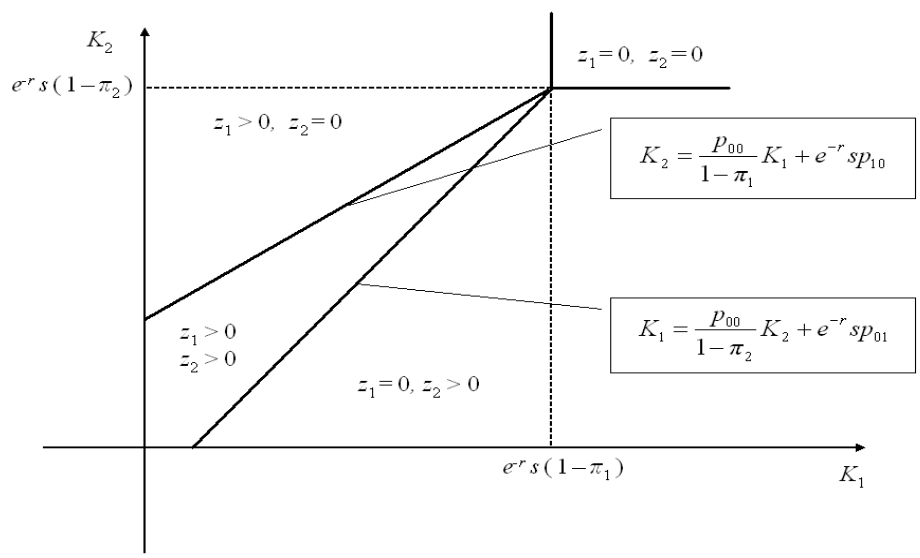
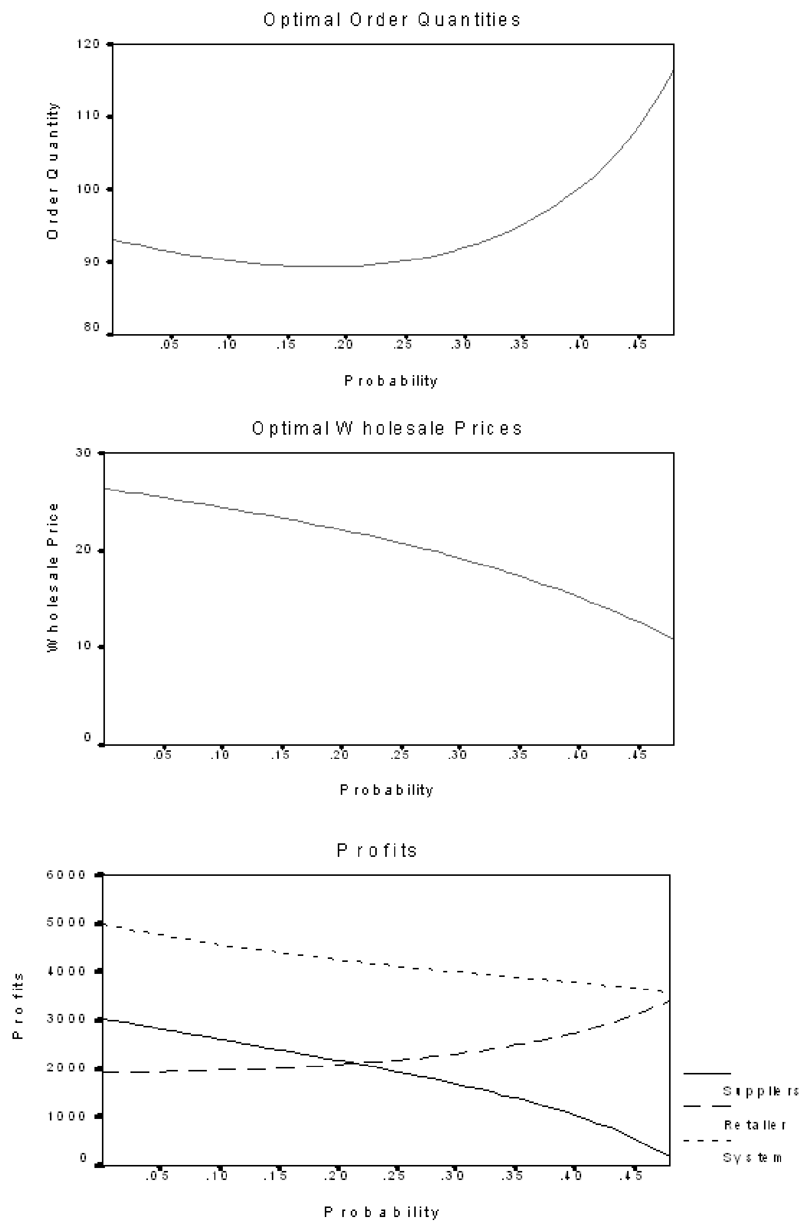


Figure 4: Symmetric Equilibrium Results for Exponential Demand

The figures show the symmetric equilibrium retailer order quantities, wholesale prices and profits as a function of the probability parameter, p_{00}, that controls default correlation. The case parameters are s=100; c=30; r=0.10. Exponential demand with mean 150 units.



Appendix. Proofs.

Proof of Proposition 3.

Recall that $A(z,\beta) = \beta\overline{G}(z\beta)$ and $A_\beta(z,\beta) = \overline{G}(\beta z)\,[1 - h(\beta z)]$. Because the demand distribution is IGFR and $\beta \le 1$, $h(z\beta) \le h(z) < 1$ for all $z < \overline{z}$ and for all β. Therefore, for all $z \le \overline{z}$, $A_\beta(z,\beta) > 0$ and, hence $A(z,\cdot)$ in an increasing function. Thus, for all $z \le \overline{z}$ as $\beta \downarrow_{st}$, $E[A(z,\beta)]$ decreases. In addition, observe that $E[A(z,\beta)]$ is decreasing in z for any given random variable β. Therefore, by the proposition hypothesis, the optimal order quantity corresponding to β_{max}: $z^{central}(\beta_{max}) < \overline{z}$. It follows that for all $\beta_1 <_{st} \beta_2 <_{st} \beta_{max}$, $z(\beta_1) < z(\beta_2) < z(\beta_{max})$. This proves the first part of Proposition 3.

Next, observe that $Pr(D < z^{central}\beta) = E[G(z^{central}\beta)]$. Because $G(z\beta)$ is an increasing function of β for all z and because $z^{central}$ decreases as credit risk increases, it follows that, as $\beta \downarrow_{st}$, $Pr(D < z^{central}\beta)$ decreases. ■

Proof of Lemma 1.

As $z \to +\infty$, by the Monotone Convergence Theorem, $S(z) \to -\infty$. Therefore, there exists $\widehat{z}$ such that for all $z > \widehat{z}$, $S(z) < 0$. Hence, we can restrict the search for an optimal z to the interval $[0, \widehat{z}]$. Function $S(\cdot)$ is bounded from above on this interval and hence, achieves the maximum. The maximum satisfies the first order conditions (11). ■

Proof of Theorem 1.

Consider the first order condition (13) that determines the optimal order quantity z^*. As π increases, the right hand side of the expression (13) increases. Because left hand side of the expression (13) is nondecreasing in z it follows that z^* is decreasing in π.

From expression (12) for supplier's profit, we see that for all z, $S(z)$ is decreasing in π. It follows that the optimal supplier's profit $S(z^*)$ is decreasing in π.

Finally, if the supplier charges a wholesale price of $K^* = e^{-r}(1-\pi)s\overline{G}(z^*_\pi)$, then the retailer's profit

$$R_\pi(z) = e^{-r}(1-\pi)s\left[E\min(D,z) - \overline{G}(z^*_\pi)z\right]$$

is decreasing in π for all z. Hence, $R^* = R(z^*)$ is decreasing in π. ■

Proof of Proposition 4.

Observe that, by the proposition hypothesis, it is not optimal for the retailer to order amounts from the suppliers that add up to a quantity lower than D. Therefore, we restrict the search for the optimal order quantities to $z_1^* + z_2^* \geq D$ and $z_i^* \leq D, i = 1, 2$. Using equation (19) we derive the following expression for the retailer's profit:

$$R(z_1, z_2) = (e^{-r} sp_{01} - K_1)z_1 + (e^{-r} sp_{10} - K_2)z_2 + p_{00}D.$$

The three cases now follow easily:

If $K_1 \leq e^{-r} sp_{01}$ and $K_2 \leq e^{-r} sp_{10}$, then order quantities (D, D), maximize retailer's profits.

If $K_1 \leq e^{-r} sp_{01}$ and $K_2 > e^{-r} sp_{10}$, then the optimal order quantities are $(D, 0)$.

If $K_1 > e^{-r} sp_{01}$ and $K_2 \leq e^{-r} sp_{10}$, then the optimal order quantities are $(0, D)$.

Suppose $K_1 > e^{-r} sp_{01}$ and $K_2 > e^{-r} sp_{10}$. Then the retailer would like to order as little as possible from the suppliers subject to the constraint $z_1 + z_2 \geq D$. Therefore, the retailer will order D from one of the suppliers and 0 from the other unless she is indifferent between the two [which occurs when $K_2 = K_1 + e^{-r} s(\pi_1 - \pi_2)$]. ■

Proof of Proposition 6.

From the first order conditions, $(z_1, 0)$ is the optimal retailer's response if

$$\left.\frac{\partial R}{\partial z_1}\right|_{z_2=0} = 0 \quad \text{and} \quad \left.\frac{\partial R}{\partial z_2}\right|_{z_2=0} \leq 0,$$

Or equivalently,

$$\begin{cases} e^{-r} s(1 - \pi_1)\overline{G}(z_1) = K_1 \\ e^{-r} s \left[p_{10} + p_{00}\overline{G}(z_1)\right] \leq K_2. \end{cases}$$

Equivalently, $e^{-r} s(1 - \pi_1)\overline{G}(z_1) = K_1$ and $K_2 \geq \frac{p_{00}}{1-\pi_1} K_1 + e^{-r} sp_{10}$. The proof for the remaining cases is similar. ■

Proof of Corollary 1.

Without loss of generality, assume that $i = 1$. By the inverse function theorem, the solution of the system (23c) is unique and differentiable. The conclusion follows from an observation that system of equations (23c) is equivalent to the system (23a) when $K_1 = \frac{(1-\pi_1)}{p_{00}} \left[K_2 - e^{-r} sp_{01}\right]$ and is equivalent to the system (23b) when $K_1 = \frac{p_{00}}{1-\pi_2} K_2 + e^{-r} sp_{10}$. ■

Proof of Proposition 8.

Because the equilibrium is symmetric, the equilibrium order quantity $z_1 = z_2 = z > 0$. Thus, we consider the supplier's profit function over the region where both order quantities are positive.

For supplier 1: $\max_{L(K^*)\le K_1\le R(K^*)}(K_1-c)z_1(K_1,K^*)$,

where $z_1(K_1,K^*)$ satisfies the system of equations (23c), $R(K^*)=\frac{p_{00}}{1-\pi_2}K^*+e^{-r}sp_{10}$, and $L(K^*)=\frac{1-\pi_1}{p_{00}}\left(K^*-e^{-r}sp_{01}\right)$. For this optimization problem we can change the variable from K_1 to z_1,z_2, as long as (23c) is satisfied. Then the optimization problem becomes:

$$\max_{z_1,z_2:L(K^*)\le K_1(z_1,z_2)\le R(K^*)}\left\{e^{-r}s\left[p_{01}\overline{G}(z_1)+p_{00}\overline{G}(z_1+z_2)\right]-c\right\}z_1,$$

subject to $\quad e^{-r}s\left[p_{10}\overline{G}(z_2)+p_{00}\overline{G}(z_1+z_2)\right]=K^*$.

Taking the Lagrangian:

$$\begin{aligned}\max_{z_1,z_2:L(K^*)\le K_1(z_1,z_2)\le R(K^*)}&\left\{e^{-r}s\left[p_{01}\overline{G}(z_1)+p_{00}\overline{G}(z_1+z_2)\right]-c\right\}z_1-\\&-\lambda\left\{e^{-r}s\left[p_{10}\overline{G}(z_2)+p_{00}\overline{G}(z_1+z_2)\right]-K^*\right\},\end{aligned}$$

the first order necessary conditions for an interior maximum point are

$$\begin{aligned}&e^{-r}s\left[p_{01}\overline{G}(z_1)+p_{00}\overline{G}(z_1+z_2)\right]-c-e^{-r}s\left[p_{01}g(z_1)+p_{00}g(z_1+z_2)\right]z_1+\\&+\lambda e^{-r}sp_{00}g(z_1+z_2)=0,\\&-e^{-r}sp_{00}g(z_1+z_2)z_1+\lambda e^{-r}s\left[p_{10}g(z_2)+p_{00}g(z_1+z_2)\right]=0,\\&e^{-r}s\left[p_{10}\overline{G}(z_2)+p_{00}\overline{G}(z_1+z_2)\right]=K^*.\end{aligned}$$

After eliminating λ from the first two equations we obtain:

$$\begin{aligned}&\left[p_{01}\overline{G}(z_1)+p_{00}\overline{G}(z_1+z_2)\right]-\left[p_{01}g(z_1)+p_{00}g(z_1+z_2)\right]z_1+\\&+\frac{p_{00}^2g^2(z_1+z_2)z_1}{p_{10}g(z_2)+p_{00}g(z_1+z_2)}=\frac{c}{e^{-r}s},\quad\text{and}\\&e^{-r}s\left[p_{10}\overline{G}(z_2)+p_{00}\overline{G}(z_1+z_2)\right]=K^*.\end{aligned}$$

For a symmetric equilibrium, $z_1=z_2=z$. Hence, the equilibrium order quantity must satisfy

$$p_{01}\overline{G}(z)[1-h(z)]+p_{00}\overline{G}(2z)\left[1-\frac{1}{2}h(2z)\right]+\frac{p_{00}^2g^2(2z)z}{p_{10}g(z)+p_{00}g(2z)}=\frac{c}{e^{-r}s},$$

where $h(z)=z\frac{g(z)}{\overline{G}(z)}$ is the generalized failure rate function. The symmetric equilibrium order quantity is related to the symmetric equilibrium wholesale prices by

$$e^{-r}s\left[p_{10}\overline{G}(z)+p_{00}\overline{G}(2z)\right]=K^*. \qquad \blacksquare$$

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