Option Pricing with Downward Sloping Demand Curves: The Case of Supply Chain Options*

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Abstract

This article investigates the role of option contracts in a supply chain when the demand curve is downward sloping. We consider call (put) options that provide the retailer with the right to reorder (return) goods at a fixed price. We show that the introduction of option contracts causes the wholesale price to increase and the volatility of the retail price to decrease. In general, options are not zero sum games. Conditions are derived under which the manufacturer prefers to use options. When this happens the retailer is also better off if the uncertainty in the demand curve is low. However, if the uncertainty is sufficiently high, the introduction of option contracts alters the equilibrium prices in a way that hurts the retailer.

(*Real Options; Downward Sloping Demand Curve; Stackelberg Games; Supply Chain Contracts*)

Many studies have been performed in industries such as apparel, sporting goods and toys, where there are long lead times, short selling seasons and high demand uncertainties. For these problems one avenue of research is concerned with the design of contracting relationships that provide retailers with flexibility in responding to unanticipated demand and prices over the sales season. This paper examines such contracting arrangements in a supply chain setting consisting of an upstream party (the manufacturer) whose only access to the product market is via a single downstream party (the retailer). To manage the risk of inventories associated with uncertain demand, it is fairly common for the manufacturer to make available to the retailer an array of products, including *reordering* contracts, or call options, that allow the retailer to purchase additional goods at a predetermined time for a fixed price, and *return* contracts, or put options, that allow the retailer to return unsold goods at a predetermined salvage price. By purchasing inventory, together with a portfolio of these call and put options, the retailer has more choices that allows a strategy to be put into place to maximize its profits. The manufacturer's goal is to design the terms of the reordering and return option contracts and establish their prices, together with the wholesale price, so as to induce the retailer to take optimal actions that best serve the manufacturer's interests.

Our primary objective is to explore how these options are valued, and how they assist in coordinating the supply chain. The valuation of the options is made difficult because the underlying good is supplied by a monopolist, and the uncertain demand curve is downward sloping. The usual approach in pricing real options follows the Black-Scholes (1973) and Merton (1973) paradigm, in which the underlying price is unaltered by the introduction of the option. This is sensible, since options are redundant contracts that can be replicated by dynamic self financing trading schemes in the underlying asset and in riskless bonds, and hence should not affect the underlying price. In our model, however, the underlying is a good supplied by a monopolist, and its wholesale price is affected by whether or not she chooses to offer options to the retailer. If options are introduced, then the retailer's sequence of actions are altered. These actions include purchasing decisions, and retailer pricing decisions contingent on demand realizations. This in turn has a feedback effect on the profit function of the monopolist. Indeed, because she can anticipate the optimal response of the retailer she can adjust the underlying wholesale and option prices so as to best meet her interests. In a complete market, which we will assume, every set of cash flows can be uniquely priced, so the monopolist can establish the program that maximizes value, without having to explicitly model preferences.[1]

In this paper we are particularly interested in how wholesale and retail prices adjust after the introduction of options. We are also interested in establishing the pricing mechanism that the manufacturer uses for the options. Indeed, our problem environment is set up so that

[1]If the assumption of complete markets is onerous, then our analysis is still valid under the assumption of risk neutrality.

we can closely examine the pricing of option contracts in a downward sloping demand curve environment.

In our framework, the benefit of the option program to the manufacturer is influenced by the uncertainty in the demand curve and the cost of filling expedited orders that arise when options are exercised late in the season. The manufacturer will only introduce options if they increase profits. Expanding the investment opportunity set, however, may not necessarily improve the condition of the retailer. Indeed, we show that when the demand curve is very uncertain, then the retailer is worse off when options are introduced. This result may, at first glance, be a bit surprising, since one might surmise that when uncertainty is high, the retailer will be more inclined to use option contracts. Of course, the manufacturer recognizes that this is the case, and adjusts the wholesale and option prices accordingly. The option contracts are not zero sum games between the manufacturer and the retailer, and, as we shall see, there are cases where both parties benefit. If the retailer is worse off, then it appears that he could threaten the manufacturer by not using options at all. However, as we will see, such a threat is not credible, since, in the presence of a new wholesale price, the retailer will just be making himself worse off by not using options.

The equilibrium wholesale price can be affected by the introduction of reorder options. However, once introduced, we show that the wholesale price will not readjust again if the manufacturer chooses to introduce return options, provided the range of strike prices are curtailed. As a result, if call options are present, then the price of put options follow directly from put-call parity.

The paper proceeds as follows. In section 1 we review related literature in finance and supply chain management. In section 2 we list the basic assumptions and establish the decision and pricing problems. We formulate the problem for the case where the manufacturer is considering the use of reordering options. Later on we show how the solutions to this problem can be used to solve for the return option policies and their prices. In section 3 we establish the benchmark case, where the manufacturer does not provide the retailer with any option contracts. This situation is of some interest in its own right. Specifically, after purchasing inventory, the retailer retains the option to withhold some product from the market. This *retention* option is shown to be valuable, especially when there is large uncertainty in the demand curve. In section 4 we consider the case when the retailer is allowed to purchase reorder options together with inventory purchases. We closely investigate how the equilibrium wholesale price and retail prices are affected by the introduction of such contracts. In section 5 we isolate conditions under which both parties benefit from the option program. We also identify cases where the option program benefits the manufacturer, but not the retailer. Section 6 investigates the consequences to the supply chain if the manufacturer and supplier act as a single firm. This analysis provides us with a useful benchmark that allows us to explore the potential of option contracts in coordinating the channel. Section 7 extends the analysis by allowing the manufacturer to use put options as well as call options. Section 8 concludes.

1 Literature Review

In the financial options literature, there is a large number of studies that have investigated how listed option contracts could alter the dynamics of asset prices. The popular view that derivatives were beneficial because they expanded the investment opportunity set, allowing traders to more precisely mold returns in accordance with their beliefs and preferences, lacked rigor, because it ignored potentially harmful feedback effects. Theoretical models that explore these feedback effects draw different conclusions. Detemple and Selden (1988) as well as Grossman (1988) identify different sets of conditions whereby the introduction of options leads to a reduction in volatility of prices, whereas Stein (1989) identifies conditions that lead to increased volatility. The empirical evidence, provides modest support for options to lead to reductions in volatility.[2] The empirical literature also concludes that the listing of call (put) options is associated with positive (negative) excess abnormal returns on the underlying asset, while the simultaneous listing of both has little effect.[3] The results of our analysis in the real options setting echo some of the results from this literature in the sense that we find the introduction of supply chain options is accompanied by price effects, both at the wholesale and at the retail levels. In particular, we show that retail prices are less volatile then they would be if no options were utilized. We also identify conditions under which retail prices may increase or decrease. Finally, with regard to wholesale prices, we show that they will either stay the same or increase.

In the operations management literature there is a large literature that examines the trade offs that exist among manufacturers and retailers for bearing inventory risk. One way for the retailer to manage demand uncertainty is to transfer it to the manufacturer who may be better equipped to absorb it. Contracts are designed not only to meet the risk aversion needs of the parties in the supply chain, but also to help coordinate the channel. Examples of contracting relationships include Eppen and Iyer (1997), who consider back up agreements, Serel, Dada and Moskowitz (2001), who consider capacity guarantees, and Brown and Lee (1997), who consider pay-to-delay capacity reservation contracts. Barnes-Schuster, Bassok and Anupindi (2000), show that all of these contracts can be viewed as special cases of option contracts that permit expedited orders. For excellent reviews of this literature see Anupindi and Bassok (1999), Lariviere (1999) and Tsay Nahmias and Aggrawal (1999). Within this literature, an important consideration is channel coordination. Oftentimes this can be accomplished through non linear pricing as in Iyer

[2]Examples of studies include Damodaran and Lim (1991) and the references in Damodaran and Subrahmanyam (1992).

[3]Surprisingly, Conrad (1989) finds that the abnormal returns are generated around the listing date, rather than the earlier announcement date.

and Bergen (1997). In other cases, coordinating the supply chain is accomplished by modifying the contract between wholesalers and retailers. For example, Barnes-Schuster, Bassok, and Anupindi (2000), show that channel coordination could be achieved by modifying the nature of the strike price in their contracting arrangements. Other examples of contracting mechanisms that induce channel coordination include Pasternak (1985), who considers product returns, and Taylor (2001), who considers combination of returns and retailer price protection. In these studies the typical methodology employed builds off the classical newsvendor model where inventory levels are established before demand is realized. Finally, in a recent paper, Cheng, Ettl, Lin, Schwarz and Yao (2003) consider option programs in a manufacturer-retailer supply chain, where the wholesale price is assumed to remain fixed before and after options are introduced, the retail price is constant and the demand random. They derive option pricing expressions and explore the effects of introducing options on the channel profits.

Our study is related to the above papers in that we consider contracting arrangements in a supply chain in a newsvendor environment. Our approach, however differs in that we do not take the retail price and the demand distribution as given; rather, we assume demand distributions are influenced by pricing decisions. In particular, we assume the retailer is faced with an uncertain downward sloping demand curve, and hence has to establish a retail price conditional on previous inventory decisions. In this regard our analysis relates to an extended single agent newsvendor problem discussed by Petruzzi and Dada (1999), who permit stocking policies and prices to be determined simultaneously. It also relates to Ritchken and Tapiero (1986) who investigate option contracts for purchasing decisions, in which price and quantity uncertainty are correlated. Both these studies deal with single agents, whereas our analysis deals with two agents in the presence of an uncertain linear downward sloping demand curve.

Several studies have investigated contingent claim pricing in the presence of a downward sloping demand curve. Triantis and Hodder (1990) examine the pricing of complex options that arise in a flexible production system which allows the firm to switch its output mix over time, in the presence of demand curves which are stochastic and downward sloping. Pindyck (1989) and He and Pindyck (1989) also incorporate downward sloping demand curves to examine capacity choice decisions in a real options framework. Our study differs from these in that we have a specific supply chain in which the optimal contracts can be designed and priced through the appropriate application of Stackelberg game theory.

2 The Basic Model

We consider a simple supply chain consisting of a single manufacturer who produces a product and sells it at date 0 to a retailer at a wholesale price of S_0 per unit. At date 0, the future date 1 demand curve is uncertain. Without any risk management contracts, the retailer is faced with bearing this uncertainty. The manufacturer assists in risk sharing by allowing the retailer to supplement inventory decisions with option purchases. Each option contract provides the retailer with the right to purchase an additional unit at date 1, after uncertainty is revealed, for a predetermined price of X dollars.

Let I and U be the number of goods and options purchased at date 0 by the retailer. The price of each good is S_0 and the price of each option is C_0. The retailer's problem is to determine the optimal mix of inventory and options, given their prices. The manufacturer's problem is to establish the optimal prices of the product and the option that induces the retailer to take rational decisions that further the interests of the manufacturer. The manufacturer is particularly keen to assess the value to the firm of issuing option contracts.

We assume the demand in period 1 is linear in the quantity of goods released into the market. Denoting the retail price in period 1 by S_1, the inverse demand curve is:

$$S_1 = \alpha - \delta Q \tag{1}$$

where Q is the total amount of inventory released into the market. Here α is stochastic, and if $\delta > 0$, α/δ can be viewed as the maximum potential size of the market.

At date 1 the uncertainty in the market size factor, α, is resolved, and the retailer responds by establishing how many units of inventory to release into the market and how many options to exercise. Let $0 \leq q \leq I$ be the number of units of inventory released and let $0 \leq v \leq U$ be the number of options exercised. The total number of items released into the market place is $Q = q + v$. We assume that any excess inventory that is held back has no salvage value. Let R_1 be the net cash flow that the retailer makes in period 1. Then

$$R_1(q, v|I, U, \alpha = a) = (q + v)(a - \delta(q + v)) - vX \tag{2}$$

In period 1, the retailer chooses q and v to maximize equation (2) subject to the inventory and option constraints. If the exercise price X is too high, then the retailer, who has exclusive rights to the market, has incentives to renegotiate the price. To ensure that the option contract is credible, the manufacturer must therefore set the exercise price no higher than the marginal cost of expedited production in period 1.[4]

Now, consider the retailer's decision at date 0. Let R_0 be the net present value associated with purchasing I units of inventory and U options at date 0 and optimally managing the project in period 1. Then

$$R_0(I, U) = -IS_0 - UC_0 + E[R_1 D_1] \tag{3}$$

where D_1 is the state dependent stochastic discount rate, commonly referred to as the pricing kernel.

[4]This assumes that the manufacturer builds to order and is faced with a linear cost structure. We will discuss the cost structure for the manufacturer shortly.

We assume that the uncertainty in the demand curve is represented by a Bernoulli process. In particular:

$$\alpha = \begin{cases} a_H & \text{with probability} \quad p \\ a_L & \text{with probability} \quad 1-p \end{cases} \tag{4}$$

We also assume that there exists a traded security that pays out a_H dollars in the high (H) state, and a_L dollars in the low (L) state. The price of this security at date 0 is A_0. In addition, a riskless bond exists that pays out \$1 in period 1. Its current price is $B_0 < 1$. The existence of traded securities that span the uncertainty in the demand curve allows the pricing kernel to be uniquely determined.[5] The assumption of complete markets allows us to perform the valuations without regard to the specific risk preferences of the retailer and manufacturer. In addition, the valuation can proceed, even if there is not a consensus on the value of p.

Let e_H (e_L) be the Arrow Debreu state price corresponding to a \$1.0 payout only in the high (low) state and \$0 payout otherwise. Clearly:

$$\begin{aligned} A_0 &= a_H e_H + a_L e_L \\ B_0 &= e_H + e_L. \end{aligned}$$

Given the state prices we have:

$$R_0(I,U) = -IS_0 - UC_0 + e_H R_1(q_H^*, v_H^* | I, U, a_H) + e_L R_1(q_L^*, v_L^* | I, U, a_L), \tag{5}$$

where $R_1(q_H^*, v_H^*|I,U,a_H)$ and $R_1(q_L^*, v_L^*|I,u,a_L)$ are the maximum values of R_1 in equation (2) with $\alpha = a_H$ and $\alpha = a_L$ respectively.

The retailer's time 0 optimization problem is given by

$$Max_{I \geq 0, U \geq 0} R_0(I,U) \tag{6}$$

The objective of the manufacturer is to maximize value by appropriately determining the wholesale price, S_0, and the charge for each option, C_0. Let M_0 be the net present value associated with this specific project. Then.

$$M_0 = IS_0 + UC_0 + e_H[Xv_H^*] + e_L[Xv_L^*] - K_0(I) - e_H K_1(v_H^*) - e_L K_L(v_L^*). \tag{7}$$

In this equation, $K_0(I)$ represents the total cost of making up I units for delivery at date 0, and $K_1(v)$ represents the cost of expediting an additional v units on date 1. For simplicity, we shall assume that the costs are linear, i.e., $K_0(q) = K_0 q$ and $K_1(v) = K_1 v$ where K_0 and K_1 are known constant marginal production costs. To ensure that the manufacturer prefers receiving committed orders at date 0 rather than expedited orders, at date 1, we require

$$K_1 B_0 > K_0. \tag{8}$$

[5]For an in depth discussion on this point see Duffie (1996).

Further, to ensure that the manufacturer does not find it advantageous to build inventories as a contingency against *possible* orders arriving in period 1, we require:

$$K_0 \geq K_1 e_j, \text{ for } j = L, H. \tag{9}$$

Finally, we assume that $K_0 < A_0$, since A_0 represents the value at date 0 of a unit of product sold at the highest possible retail price at date 1.

The manufacturer's problem is to establish the wholesale price, S_0 and the reorder option price, C_0, such that:

$$Max_{S_0,C_0} M_0(S_0, C_0) \tag{10}$$

given the fact that the retailer responds optimally for each action. This formulation is a standard Stackelberg game with complete information, with the manufacturer being the leader.

In the sequel we often find it helpful to represent the uncertainty in the demand curve using the following reparametrization. Let μ_A and σ_A^2 represent the mean and variance of the intercept term of the demand curve under the risk neutral measure.[6] It is easy to find that

$$\begin{aligned}
\mu_A &= \frac{A_0}{B_0} \\
\sigma_A^2 &= \frac{e_H e_L}{B_0^2}(a_H - a_L)^2,
\end{aligned}$$

Also let $\rho = \frac{e_L}{e_H}$. Then we can express a_L, a_H, e_L, e_H in terms of $\mu_A, \sigma_A, B_0, \rho$ as follows

$$a_H = \frac{A_0}{B_0} + \sqrt{\rho}\sigma_A \tag{11}$$

$$a_L = \frac{A_0}{B_0} - \frac{1}{\sqrt{\rho}}\sigma_A \tag{12}$$

$$e_H = \frac{B_0}{1+\rho} \tag{13}$$

$$e_L = \frac{B_0 \rho}{1+\rho} \tag{14}$$

3 Pricing with No Supply Chain Option Contracts

In this section we analyze the supplier and retailer's equilibrium strategies when option contracts are not offered. In particular, we examine how the equilibrium wholesale price S_0^* and order quantity I_0^* vary with the volatility σ_A of the demand curve, while the other parameters (μ_A, B_0 and ρ) are kept constant. In this context the deterministic case $\sigma_A = 0$ is interesting, because it can be used as a benchmark to explore the effects of uncertainty. Let S_{det} and I_{det} denote the

[6] Under this measure the expected growth rate of all traded securities equals the risk free rate. In this model it is determined by the two risk-neutral probabilities $q_j = e_j/B_0, j = L, H$.

equilibrium wholesale price and order quantity, respectively, when $\sigma_A = 0$. It is easy to show that:

$$\begin{aligned} S_{det} &= \frac{A_0 + K_0}{2} \\ I_{det} &= \frac{A_0 - K_0}{4\delta B_0}. \end{aligned}$$

It is also clear that in this case the quantity released in the market at date 1 is equal to $q_{det} = I_{det}$, because with no uncertainty the retailer orders at date 0 exactly the quantity that is optimal to release in the market at date 1.

When the volatility is not zero, an important role is played by the retailer's ability to withhold some product from the market if the demand revealed at date 1 is low. This ability to control the quantity sold is called a *retention option*. It is intuitive that the retention option should become more valuable when σ_A increases. The following result shows the equilibrium price and quantities when the volatility is no longer zero.

Proposition 1 *When no supply chain options are offered:*

(i) The equilibrium wholesale price depends on the volatility σ_A. In particular,

$$S_0^* = \begin{cases} S_{det}, & \text{if} \quad \sigma_A \leq \eta \\ S_{det} - \frac{e_L}{2}(\frac{A_0}{B_0} - \frac{\sigma_A}{\sqrt{\rho}}), & \text{if} \quad \sigma_A > \eta \end{cases} \tag{15}$$

where

$$\eta = \sqrt{\rho}\left[\frac{1+\rho}{1+\rho+\sqrt{1+\rho}}\frac{K_0}{B_0} + \frac{\sqrt{1+\rho}}{1+\rho+\sqrt{1+\rho}}\frac{A_0}{B_0}\right]. \tag{16}$$

(ii) The equilibrium order quantity at date 0 is

$$I_0^* = \begin{cases} I_{det}, & \text{if} \quad \sigma_A \leq \eta \\ I_{det} + \frac{\rho}{4\delta}(\frac{\sigma_A}{\sqrt{\rho}} - \frac{K_0}{B_0}), & \text{if} \quad \sigma_A > \eta \end{cases} \tag{17}$$

(iii) The retailer exercises the retention option only when the volatility is high. Specifically, the quantities sold at date 1 are:

$$q_L^* = \begin{cases} I_{det}, & \text{if} \quad \sigma_A \leq \eta \\ I_{det} + \frac{1}{2\delta}(\frac{S_{det}}{B_0} - \frac{1}{\sqrt{\rho}}\sigma_A) & \text{if} \quad \sigma_A > \eta \end{cases} \tag{18}$$

$$q_H^* = \begin{cases} I_{det}, & \text{if} \quad \sigma_A \leq \eta \\ I_{det} + \frac{\rho}{4\delta}(\frac{\sigma_A}{\sqrt{\rho}} - \frac{K_0}{B_0}) & \text{if} \quad \sigma_A > \eta \end{cases} \tag{19}$$

Proof: See Appendix

Proposition 1 implies that if the volatility is lower than a threshold value, η, the equilibrium is identical to that of the deterministic case, in that the wholesale price, the order quantity and the quantities released to the market are constant with respect to σ_A. When the uncertainty is relatively small, the retailer ensures that enough inventory is purchased so that in the low state the optimal quantity is released, while in the high state there is only minor regret. In other words, the retailer has little incentive to build excess inventory, beyond what is optimal for the lower state, since the additional revenues captured by having such inventory available, if the high state occurs, does not offset the additional costs of purchasing inventory at date 0, which may go unused. The manufacturer, of course, recognizes that the retention option is not worth much, and hence charges a price, based on the premise that the retailer will rationally commit to selling all inventory purchased.

As the volatility expands, the consequences are no longer minor. Eventually, the retailer will want to order more than is required for the lower state, but not enough to cover the optimal amount in the high state. If the low state occurs, the retailer has to establish the amount to be released into the market. By retaining some of the units, the retailer ensures a higher per unit retail price. However, since the units are costly, the retailer will take this into account when he makes the ordering decision, and will be less likely to order too much. The manufacturer recognizes the importance of the retailer's retention option, and induces the retailer to purchase more inventory, by offering a lower wholesale price. The retailer responds to the lower price by purchasing more units than necessary for the lower state, but not quite enough for the high state. Since the gap between states is sufficiently large, and since the per unit cost is low, the retailer is prepared to withhold some inventory in the low state, rather than releasing it into the market.

The retail prices, $S_j = a_j - \delta q_j, j = H, L$, of course, depend on the state and on the amount released. The range of retail prices, $R_{\text{No Opt.}} = S_H - S_L$, obtained using equations (18), (19), (11) and (12) simplifies to:

$$R_{\text{No Opt.}} = \begin{cases} \frac{1+\rho}{\sqrt{\rho}}\sigma_A & \text{if} \quad \sigma_A \leq \eta \\ \frac{2+3\rho}{4\sqrt{\rho}}\sigma_A + \frac{A_0+K_0(1+\rho)}{4B_0} & \text{if} \quad \sigma_A > \eta. \end{cases} \tag{20}$$

We will use this equation later on to examine whether the introduction of options affects the volatility of retail prices.

Given the optimal policy for the manufacturer, we can compute the optimal profit of the manufacturer and examine the implications of the policy for the profits associated with the optimal response of the retailer.

Proposition 2 *(i) The net present value for the manufacturer given that no option contracts* *are used, M_0^*, equals:*

$$M_0^* = \begin{cases} \frac{(A_0-K_0)^2}{8\delta B_0} & \textit{if} \quad \sigma_A \leq \eta \\ \frac{(a_H e_H - K_0)^2}{8\delta e_H} & \textit{if} \quad \sigma_A > \eta, \end{cases} \tag{21}$$

(ii) The net present value for the retailer given that no option contracts are used, R_0^*, equals:

$$R_0^* = \begin{cases} \frac{(A_0-K_0)^2}{16 B_0 \delta} & \textit{if} \quad \sigma_A \leq \eta \\ \frac{a_H^2 e_H^2 + 4a_L^2 e_L e_H - 2K_0 a_H e_H + K_0^2}{16\delta e_H} & \textit{if} \quad \sigma_A > \eta. \end{cases} \tag{22}$$

Proof: The expressions follow directly by substituting the manufacturer's and retailer's optimal policies from proposition 1 into the corresponding profit expressions.

To illustrate the results, assume the bond price, $B_0 = 0.8$, the asset price, $A_0 = 20$, the ratio of state prices, $\rho = 4$, and the slope of the demand curve, $\delta = 1$. Further, assume the per unit cost, $K_0 = 10$. With these values, $S_{det} = 13$, and $I_{det} = 1.875$. The volatility can range from 0 to 40. The critical cut off point, $\eta = 29.6$. Figure 1 shows how the wholesale price, order quantity and profits for the manufacturer and retailer are affected as the volatility of the intercept of the curve, σ_A, increases from 0 to its maximum value. Notice that the equilibrium wholesale price remains constant until the value of η is attained, at which point there is a discontinuity. Thereafter, the price is increasing in volatility. The optimal order quantity is flat until η is reached, at which point there is a jump. The optimal profits for the manufacturer increase with volatility, beyond η. For the retailer, the profits are quadratic in volatility, and not necessarily increasing.

Figure 1 Here

4 Pricing with Supply Chain Option Contracts

We now reconsider the above problem, but this time we also assume that the manufacturer offers a reorder option, in which each contract provides the retailer with the option of purchasing one extra unit at a predetermined price of X. Assume the unit cost of this option is C_0 and the wholesale price for the product is S_0.

4.1 The Retailer's Response

To solve the Stackelberg game we first assume the manufacturer's wholesale price, S_0, and option price, C_0, are given. Conditional on these prices, we then solve the retailer's problem. This is a standard 2-period problem, solved by first conditioning on the retailer's ordering decision, (I, U), in period 0, and then establishing the optimal number of options to exercise and inventory to release in period 1, once the demand curve is revealed. In period 1, the retailer maximizes the following function:

$$R_1(q, v|I, U, a) = (q + v)(a - \delta(q + v)) - Xv.$$

Lemma 1 below identifies the optimal strategy for the retailer conditional on (I, U) and on the realized demand curve.

Lemma 1 *At date 1, the retailer's optimal policy has the following form:*

1. *If $I > \frac{a}{2\delta}$ then the optimal number of units to sell is $q^* = \frac{a}{2\delta}$ with $v^* = 0$.*
2. *If $\frac{a-X}{2\delta} < I < \frac{a}{2\delta}$ then it is optimal for the retailer to sell all inventory, but not to exercise any options.*
3. *If $0 < I < \frac{a-X}{2\delta}$ then it is optimal to sell all inventory and to exercise $v^* = Min[U, \frac{a-X}{2\delta} - I]$ options.*

Proof: See Appendix

Figure 2 shows two possible sets of regions of (I, U) over which the optimal responses by the retailer in period 1 can be identified. The first panel, corresponds to the case, where $\frac{a_L}{2\delta} > \frac{a_h - X}{2\delta}$, or $a_H - a_L < X$. The second panel corresponds to the case where, $a_H - a_L \geq X$. In each figure, both realizations of demand are given. The retailer's optimal response in each of the indicated regions, as identified by Lemma 1, is also provided.

Figure 2 Here

As an example, conditional on being in Region R3 in Figure 1, the retailer will release all inventory and exercise no options, if the demand is low. On the other hand, if demand is high, the optimal response is to release all inventory and exercise all options. In Region R5, the retailer will also release all inventory, and exercise no options, if demand is low. However, if demand is high, not all options are exercised. Clearly, if options are costly, then at date 0 a rational retailer will not choose a point in region R5.

Having characterized the retailer's response in period 1, we now turn attention to the retailer's problem in period 0. In particular, the retailer considers the following problem:

$$R_0(I, U) = -S_0 I - C_0 U + e_L R_1(q_L^*, v_L^*|I, u, a_L) + e_H R_1(q_H, v_H^*|I, u, a_H)$$

where the values in period 1 are now know.

The next lemma identifies the regions where options may be deemed desirable by the retailer.

Lemma 2 *The only possible region where the retailer might take a positive position in options is region R3. In particular, either* $U^* = 0$*, or* $I^* \geq \frac{a_L - X}{2\delta}$ *and* $U^* \leq \frac{a_H - X}{2\delta} - I^*$.

Lemma 2 states that outside of region R3, the profits of the retailer can always be improved by purchasing stock alone and not utilizing any options. So for options to be utilized the manufacturer has to establish prices that ensures that the retailer responds optimally by choosing a point in region R3.

Towards this goal, consider the retailer's problem in region R3. In this region, we have

$$R_0(I_1^*, U_1^*) = Max[-IS_0 - UC_0 + e_L(I)(a_L - \delta I) + e_H(I+U)(a_H - \delta(I+U)) - e_H UX] \quad (23)$$

The optimal unconstrained solution for this problem is:

$$I_1^* = \frac{a_L - X}{2\delta} + \frac{C_0 - S_0 + XB_0}{2\delta e_L} \quad (24)$$

$$U_1^* = \frac{a_H - X}{2\delta} - \frac{C_0}{2\delta e_H} - I_1^*. \quad (25)$$

For this unconstrained solution to be interior to region R3, further conditions must hold. These conditions for the case where $a_H - a_L < X$ are:

$$S_0 - Xe_H - a_L e_L \leq C_0 \leq [S_0 - XB_0 + e_L(a_H - a_L)]\frac{e_H}{B_0}. \quad (26)$$

If the call is too cheap, no inventory will be held. The lower bound is therefore required to ensure that $I^* \geq 0$. If the call is too expensive, no options will be purchased. The upper bound ensures that $U^* \geq 0$.

For the unconstrained solution to be interior to region R3 for the case where $a_H - a_L > X$, in panel B of Figure 2, an additional condition is required to ensure that $I_1^* \leq \frac{a_L}{2\delta}$. In summary, for this case:

$$S_0 - Xe_H - a_L e_L \leq C_0 \leq Min\{S_0 - Xe_H, [S_0 - XB_0 + e_L(a_H - a_L)]\frac{e_H}{B_0}. \quad (27)$$

In sum, if the wholesale price, option price, and strike price, are appropriately constrained by either equation (26), for the case where $a_H - a_L < X$ or by equation (27), for the case where $a_H - a_L \geq X$, then the retailer will respond by choosing (I^*, U^*) by equations (24) and (25), which are in region R3, or by choosing U^* to be zero. Further, if the constraints do not hold, then the optimal solution will not contain any options.

4.2 The Manufacturer's Decision Using Options

Given that the retailer's optimal response involving options has been identified, we now are in a position to solve for the manufacturer's optimal policy with options. In particular, the manufacturer's profit function with options offered is:

$$M_0 = I^*(S_0 - K_0) + U^*C_0 + e_H v_H^*(X - K_1) + e_L v_L^*(X - K_L).$$

From the above analysis of the retailer's response, it follows that, when (S_0, C_0, X) satisfies either equation (26) or equation (27), depending on whether $a_H - a_L$ is lower or higher than the strike, X, then the retailer's optimal response, using options, is given by equations (24) and (25). Furthermore, for these values of I^* and U^*, the optimal number of options exercised is equal to $v_L^* = 0$ and $v_H^* = U^*$. Therefore,

$$M_0 = I^*(S_0 - K_0) + U^*(C_0 + e_H X - e_H K_1).$$

Substituting for I^* and U^* from (24) and (25) and differentiating with respect to S_0 and C_0 leads to the first order conditions, with the following solution:

$$S_0^* = S_{det} \tag{28}$$

$$C_0^* = \frac{a_H e_H + K_1 e_H - 2X e_H}{2}. \tag{29}$$

Notice, that for the option price to be nonnegative, the strike price in equation (29) must be curtailed to be less than $\frac{a_H + K_1}{2}$. Further, substituting equation (28) into the restriction, $C_0 \geq Max[S_0^* - XB_0]$ leads to another restriction on the strike. In particular, we require:

$$\frac{a_L}{2} + \frac{K_0}{2e_L} - \frac{K_1 e_H}{2e_L} \leq X \leq \frac{a_H + K_1}{2}.$$

All that remains is to check that the solution for (S_0^*, C_0^*) satisfies the feasibility conditions for the retailer to optimally respond in Region R3. Since S_0^* and C_0^* depend on the costs, K_0 and K_1, the feasibility constraints for region R3 will translate into constraints on these production costs. Substituting for C_0^* and S_0^* into equation (26), for the case where $a_H - a_L < X$, for example, leads, after some algebra to:

$$\frac{K_0 - a_L e_L}{e_H} \leq K_1 \leq \frac{K_0 + (a_H - a_L)e_L}{B_0}.$$

In addition, recall that, for economic reasons, we have assumed $\frac{K_0}{B_0} \leq K_1 \leq \frac{K_0}{e_H}$. Hence, for this case, we require:

$$\underline{k}_1 \leq K_1 \leq \overline{k}_1 \tag{30}$$

where $\underline{k}_1 = Max\left[\frac{K_0}{B_0}, \frac{K_0 - a_L e_L}{e_H}\right]$, $\overline{k}_1 = Min\left[\frac{K_0}{e_H}, \overline{K}_1\right]$, and $\overline{K}_1 = \frac{K_0 + (a_H - a_L)e_L}{B_0}$. Substituting equations (11)-(14) into the above expressions, we obtain,

$$\underline{k}_1(\sigma_A) \leq K_1 \leq \overline{k}_1(\sigma_A) \tag{31}$$

where

$$\begin{aligned}\underline{k}_1(\sigma_A) &= \frac{K_0}{B_0} + Max\left[0, \frac{(K_0 - A_0)\rho + \sqrt{\rho}B_0\sigma_A}{B_0}\right] \\ \overline{k}_1(\sigma_A) &= \frac{K_0}{B_0} + Min\left[\frac{\rho K_0}{B_0}, \sqrt{\rho}\sigma_A\right]\end{aligned}$$

For a given σ_A, equation (31) states that if the cost K_1 lies in a particular interval, then the optimal response by the retailer will be to use options since the retailer finds himself in Region R3.

For the second case, namely $a_H - a_L \geq X$, the extra constraint, $C_0^* \leq (S_0^* - Xe_H)$ translates to $K_1 e_H \leq K_0 + a_L e_L$, which is automatically satisfied. Hence, the two sets of constraints, given by equations (26) and (27), together with the economic constraints, lead to a common set of constraints on production and expediting costs as given in (31).

If the cost K_1 is fixed, then equation (31) indicates that the range of volatilities for which options will be adopted by the retailer is curtailed. In particular, inverting equation (31) gives the following interval of volatilities, for which options are used:

$$\underline{\sigma}(K_1) \leq \sigma_A \leq \overline{\sigma}(K_1), \tag{32}$$

where

$$\begin{aligned}\underline{\sigma}(K_1) &= \frac{1}{\sqrt{\rho}}(K_1 - \frac{K_0}{B_0}) \\ \overline{\sigma}(K_1) &= (K_1 - \frac{K_0(1+\rho)}{B_0})\frac{1}{\sqrt{\rho}} + \sqrt{\rho}\frac{A_0}{B_0}\end{aligned}$$

The following Proposition characterizes the optimal use of options.

Proposition 3 *(i) If the manufacturer is going to use options, then the wholesale price and option price will be set at:*

$$S_0^* = S_{det} \tag{33}$$

$$C_0^* = \frac{B_0}{2(1+\rho)}\left[\frac{A_0}{B_0} + K_1 - 2X + \sqrt{\rho}\sigma\right], \tag{34}$$

where the strike price is curtailed as:

$$\frac{K_0(1+\rho) - K_1B_0 - \sqrt{\rho}B_0\sigma_A}{2\rho B_0} \leq X \leq \frac{1}{2}\left[\frac{A_0}{B_0} + K_1 + \sqrt{\rho}\sigma_A\right] \tag{35}$$

if and only if the variance of the intercept of the demand curve, σ_A, falls in the interval: $\underline{\sigma}(K_1) \leq \sigma_A \leq \overline{\sigma}(K_1)$.

(ii) The retailer's optimal response is given by:

$$I^*_{opt} = \frac{1}{4\delta\sqrt{\rho}}\left[\overline{\sigma}(K_1) - \sigma_A\right] \tag{36}$$

$$U^* = \frac{1+\rho}{4\delta\sqrt{\rho}}\left[\sigma_A - \underline{\sigma}(K_1)\right] \tag{37}$$

$$q_L = q_H = I^* \tag{38}$$

$$v_L = 0, \quad v_H = U^*. \tag{39}$$

If σ_A does not satisfy the above constraints, then the manufacturer will not find it optimal to use options.

Proof: The results for the equilibrium prices and strike intervals follows from equations (33), (34) and (35), together with equations (11)-(14). The results for the retailer follow after substituting the wholesale and option prices back into the retailer's optimal response function given in equations (24) and (25).

Proposition 3 identifies necessary conditions that must be satisfied for the manufacturer to use options. Given these conditions hold, and that the manufacturer uses options optimally, the proposition identifies the price of the contracts. Later on, in Proposition 5, we will compare the profits for the manufacturer, with and without options, and establish that the conditions of Proposition 3 are also sufficient for the manufacturer to benefit from introducing the options program.

Without options, we have seen that the equilibrium wholesale price depends on volatility. In particular, for low volatility ($\sigma_A \leq \eta$), the price remained constant at S_{det} and for higher volatility the price was lower, depending on the volatility. In contrast, with options, the structure for the equilibrium wholesale price equals S_{det}, independent of the level of volatility. Therefore, for volatility exceeding η, the equilibrium wholesale price increases when options are introduced.

If the volatility is low, ($\sigma_A \leq \eta$), the introduction of options, does not affect the wholesale price, but does affect the retailer's response, the quantities released into the market, and the retail prices in period 1. In particular, the retailer, by purchasing options, changes inventory holdings from I_{det} to I_{opt}. The exact difference, $I_{det} - I^*_{opt}$ is given by $(\sigma_A - \underline{\sigma}(K_1))\frac{1}{4\delta\sqrt{\rho}} \geq 0$. Since in the low state, options are not exercised, the total quantity of units released into the market is lower when options are allowed. As a result, the equilibrium retail price is higher.

In the high state, however, all options are exercised, and the total amount released into the market place is $I^*_{opt} + U^*$. The difference in inventory released with options and without is $I^*_{opt} + U^* - I_{det}$ which upon substitution simplifies to $(\sigma_A - \underline{\sigma}(K_1))\frac{\sqrt{\rho}}{\delta} > 0$. Since more inventory is released with options, the equilibrium retail price in this state is lower.

Since the retail price with options in the high (low) state is lower (higher) than without options the range of retail prices is lower when options are used.

If the volatility is high($\sigma_A > \eta$), the introduction of options results in a higher wholesale price. It is this increase in price that induces the retailer to take on a position in options. In this case, the retailer's inventory holdings drop from I_{det} to a lower value. In particular, if we substitute for I_0^* and I_{opt}^* it follows that $I_0^* - I_{opt}^* \geq \frac{1}{4\delta}[K_1 - K_0] > 0$. If the low state occurs, all inventory is released and no options are exercised. Since inventory holdings are lower with options, the retail price with options will be higher. If the high state occurs, all options are exercised and the total amount of inventory released into the market for the case of options will be higher. This means the retail price for the options case will be lower. Since the range of retail prices with options is lower than the range of option prices without options, the volatility of retail prices is reduced when options are permitted.

More formally, the range of retail prices with options, R_{Opt}, is given by:

$$R_{Opt} = S_H - S_L = \frac{1+\rho}{4\sqrt{\rho}}\left[3\sigma_A + \underline{\sigma}(K_1)\right]. \tag{40}$$

Comparing equation (40) with equation (20) we see that indeed:

$$R_{No\ Opt} - R_{Opt} = \begin{cases} \frac{1+\rho}{4\sqrt{\rho}}(\sigma_A - \underline{\sigma}(K_1) & \text{if} \quad \sigma_A \leq \eta \\ \frac{2+3\rho}{4\sqrt{\rho}}\sigma_A + \frac{A_0+K_0(1+\rho)}{4B_0} & \text{if} \quad \sigma_A > \eta \end{cases} \tag{41}$$

and, since regardless of η this value is positive, options have the property that they reduce the volatility of retail prices.

Notice, that as K_1 increases, the difference between the order quantities with and without options narrows. This reflects the fact that if the cost to the manufacturer of expediting an order increases, then the benefit of options for the manufacturer decreases, and prices will be set so that the retailer does not find it beneficial to use options.

Proposition 3 states that in the low state, none of the options are exercised. In this state, the retail price is equal to $S_L = a_L - \delta I_{opt}^*$, which upon substitutions reduces to $S_L = \frac{3A_0}{4B_0} - \frac{3\sigma_A}{4\sqrt{\rho}} - \frac{1}{4\rho}(K_1 - \frac{K_0(1+\rho)}{B_0})$. From equation (35), conditions can be identified such that the lowest feasible strike price is lower than the equilibrium retail price, S_L. At first glance it appears that if these conditions are met then the retailer is allowing some "in-the-money" options to expire worthless! On the surface this is true. Recall, however, that in the low demand state, no options are exercised and all inventory is released. If one of these "in-the-money" options were exercised, then the retailer would want to sell that item. Given the downward sloping demand curve, however, the retail price of not only this item, but all items sold would be affected and overall the retailer would be worse off.

We now can compute the optimal profits for the manufacturer and retailer when options are adopted.

Proposition 4 *(i) The net present value for the manufacturer, given that option contracts are used, M^*, equals:*

$$M^* = \frac{a_H^2 e_H + a_L^2 e_L}{8\delta} - \frac{1}{8\delta e_L}[2a_L e_L K_0 - K_0^2 + 2e_H K_0 K_1 - e_H^2 K_1^2 + e_H e_L(2a_H K_1 - 2a_L K_1 - K_1^2)]. \tag{42}$$

(ii) The net present value for the retailer given that option contracts are used, R^*, equals

$$R^* = \frac{M^*}{2}. \tag{43}$$

Proof: The expressions follow directly by substituting the manufacturer's and retailer's optimal policies from Proposition 3 into the corresponding profit expressions.

5 The Value of Supply Chain Option Contracts

So far, we have not resolved the issue of whether the manufacturer should issue reorder options or not. Rather, we have only examined the consequences if option contracts were issued. In this section we analyze the impact of introducing reorder options on the profits of the manufacturer, and also we examine the implications on profits for the retailer.

Proposition 5 *If the volatility of the intercept of the demand curve, σ_A falls in the interval $\underline{\sigma}(K_1) \leq \sigma_A \leq \overline{\sigma}(K_1)$, then it is beneficial for the manufacturer to issue options.*

Proof: See Appendix

This result implies that the volatility conditions of Proposition 3 are not only necessary but also sufficient for the manufacturer to issue options. Therefore, inside the range of volatilities between $\underline{\sigma}(K_1)$ and $\overline{\sigma}(K_1)$ the value of the options program to the manufacturer is positive.

However, since the retailer can only respond to prices provided by the manufacturer, the value of the options program to the retailer could be negative. That is, the retailer could be hurt by the introduction of reorder options. In particular, when the volatility is low ($\sigma \leq \eta$) and options are offered, we have seen that their introduction leaves the wholesale price unaltered. In this case the retailer can only benefit, since the options increase his flexibility without any adverse effects. This can also be seen as a consequence of Propositions 2 and 4. Indeed, in the low volatility case it follows from the two propositions that the retailer's profit is equal to one half of the manufacturer's profit both with and without options. Since the manufacturer prefers options the retailer will prefer them as well.

On the other hand, in the case of high volatility ($\sigma > \eta$), the introduction of options offers flexibility to the retailer, but also increases the wholesale price. These two effects counteract each other and which of the two dominates depends on the situation. If the second one does, the retailer will be made worse off. Unfortunately, the retailer cannot threaten not to use options in this case. Such a threat is not credible, since it will only result in a sub-optimal response. Of course, in this case the retailer would prefer the cheaper equilibrium wholesale prices that the manufacturer offered before the option program was announced. However, given the new prices, the retailer is rationally induced into substituting some inventory for options.

To illustrate the case with options, the top panel of Figure 3 shows the behavior of stock and option prices to changes in volatility. The parameters are the same as for the no-options case. The strike price of the option is \$12, and the expedited cost, $K_1 = \$10$. The stock price remains flat, while the option price increases with volatility. The second panel shows the inventory decisions for each level of volatiltity. As volatility increases, inventory is substituted by options. The final panel shows the behavior of profits with options for both the manufacturer and the retailer.

Figure 3 Here

Figure 4 shows the value of the options program to the manufacturer and the retailer for various values of K_1. As expected from Proposition 3 the value to the manufacturer is always nonnegative, as long as the volatility is within the prescribed range.

Figure 4 Here

For the retailer there are situations that he is hurt by the options. For example in the case that $K_1 = 57.95$ we observe the following. The value for the retailer is positive for $\sigma_A \leq \eta$, as expected from the discussion above. For $\sigma_A > \eta$ and not too large, the retailer is hurt by the introduction of options. In this volatility range we have seen that the introduction of options results in a significant increase in wholesale price. On the other hand, for higher values of volatility, the price increase is less substantial and the benefits of increased flexibility become more important. As a result, as the figure shows, the value of the options to the retailer returns to the positive range.

6 Single Firm Analysis

So far we have focused on the role of reorder options by a manufacturer, who acts as a Stackelberg leader, in a supply chain with a downstream retailer who has monopoly in a marketplace and the two parties separately maximize their own profit functions. We now consider the case where the manufacturer and retailer are a single firm and thus act to maximize their joint profits. We will then compare the total profits of the manufacturer and the retailer in the previous section with those of the single firm, in order to assess how close to the coordinated channel the options program brings the two parties.

If the manufacturer and the retailer act as a single firm, then the wholesale price,S_0, the options price, C_0 and the exercise price, X, become transfer prices and do not appear in the total profit function. On the other hand, the retail price and the regular and expedited production costs remain. We now have the production problem of a firm that can produce any quantity at a cost of K_0 per unit at date 0. It then finds out the realization of the demand curve intercept (a_L or a_H), and decides whether to produce extra quantities at cost of K_1 per unit or to withhold some of the inventory at hand.

Let I denote the production at date 0 and q_L, q_H the quantities sold at date 1 if the demand is low and high respectively (if $q \leq I$ part of the original inventory is retained, while if $q > I$, then $q - I$ is the extra quantity produced at date 1. The net present value of the total profit at date 0 is

$$R_s(I, q_L, q_H) = -K_0 I + e_L \left[q_L(a_L - \delta q_L) - K1(q_L - I)^+\right] + e_H \left[q_H(a_H - \delta q_H) - K1(q_H - I)^+\right].$$

The maximization of R_s can be performed along similar lines to Sections 3 and 4, first analyzing the optimal policy at date 1 as a function of I and then optimizing over I at date 0. The main results of the analysis are summarized in the following proposition.

Proposition 6 *(i) The optimal production quantity at date 0 is equal to* $\max\{I_S^*, 0\}$*, where*

$$I_S^* = \begin{cases} 2I_{det}, & \text{if } \sigma_A \leq \frac{1}{\sqrt{\rho}}\left(K_1 - \frac{K_0}{B_0}\right) \\ 2I_{det} - \frac{1}{\delta\sqrt{\rho}}\left(\sigma_A - \frac{1}{\sqrt{\rho}}\left(K_1 - \frac{K_0}{B_0}\right)\right), & \text{otherwise} \end{cases}$$

The optimal quantities to sell at date 1 are

$$\begin{aligned} q_L^* &= I_S^* \\ q_H^* &= \max\{I_S^*, \frac{a_H - K1}{2\delta}\} \end{aligned}$$

(ii) When options are beneficial for the manufacturer, the channel profit is equal to 75% *of the optimal profit of the single firm.*

According to Proposition 6, when the volatility is low then the optimal order quantity is equal to the optimal quantity for the deterministic case $\sigma_A = 0$. In this case no production takes place at date 1 regardless of which state is realized, i.e., the emergency production option is not worth anything because of low volatility. The fact that in the deterministic case the optimal quantity to produce and sell is equal to twice as much as that in Section 4 is due to the fact that in the single firm case the double marginalization effect does not exist, which increases the production quantity. On the other hand, when volatility increases above $\frac{1}{\sqrt{\rho}}\left(K_1 - \frac{K_0}{B_0}\right)$, then the optimal policy is to produce exactly what will be sold in the low state and produce an additional quantity if the high state occurs. In this case the volatility is sufficiently high to make the emergency production option valuable. For even higher volatility values, it is optimal to produce nothing at date 0 and only produce at date if the high state occurs.

Another consequence of Proposition 6 is that by using option contracts channel coordination cannot be achieved, since only 75% of the single firm profits are realized for the channel. This suggests that there may be other types of arrangements between the manufacturer and the retailer that reach coordination. On the other hand, it is also interesting to compare the total channel profit with and without options, to explore whether and to what extent the introduction of options improves the coordination. This is illustrated in Figure 5.

Figure 5 Here

The first panel compares the total channel values with no options, to the case where options are introduced. The first best solution, consisting of the supply chain profits under a centralized ownership is also provided. The figure shows the values for the array of feasible volatility values where options make sense. In addition, the plots are for two cases that differ only in the values of K_1. In particular, the leftmost (higher) plots are for the case where $K_1 = 35.25$, and the rightmost (lower) plots are for the case where $K_1 = 57.95$. The remaining parameters are the same as in the previous examples. We observe that the values increase with volatility. Notice that the use of options has an enormous effect on coordinating the supply chain.

The second panel of Figure 5 shows how the options improve upon the no-options case. When the costs of expediting an order are sufficiently high, the percentage improvements are not increasing with volatility. The solid lines in the second figure represent the maximum percentage improvement that could be obtained, under the assumption of a single managed coordinated supply chain. The final panel of Figure 3 shows the percentage of total possible improvement that could be accomplished by using option contracts. This percentage depends critically on the level of volatility, and on the cost of expediting.

7 Pricing with Supply Chain Put Contracts

In this section we consider what happens to the price and quantity equilibrium if the manufacturer introduces put options that allow the retailer to return to the manufacturer any unsold items for a salvage price of X.

Proposition 7 *The retailer's optimal ordering policy when the manufacturer offers a wholesale price of S_0, and return options with price P_0 and exercise price X is identical to the ordering policy in a problem where the manufacturer offers a wholesale price of S_0, only call options at price C_0, with strike X, provided $C_0 = S_0 + P_0 - B_0 X$.*

Proof: The result is immediate from put-call parity.[7]

The manufacturer recognizes that the retailer can synthetically construct puts using put-call parity. As a result, solving the pricing problem for reordering options will then lead to identifying the price of return puts. There are four important implications of Proposition 7 and the results of Sections 3 and 4. First, for the problem in which the manufacturer offers only return options with salvage price X, the equilibrium wholesale price, the state dependent retail prices of the items, and the fair price of the reorder options, derived under the conditions of Proposition 3, remain unaltered. Second, provided the strike price is curtailed as in equation (35), the the optimal price the manufacturer charges for the return option, P_0, is given by

$$P_0 = e_L \left(X - \frac{a_L e_L + K_0 - K_1 e_H}{2 e_L} \right).$$

Third, for the problem in which the manufacturer offers both reorder options with exercise price X_1 and return options with salvage price X_2, the wholesale and retail prices are unchanged from what they would be if only reorder options were offered. Finally, the price of the reorder (return) option does not depend on whether the manufacturer offered the return (reorder) contract provided the strike prices are curtailed as in equation (35). Indeed, the wholesale and retail prices will remain unchanged if the manufacturer offers an array of contracts with different feasible strike prices as long as these contracts are priced accordingly.

8 Conclusion

This article has considered the problem of option pricing when the demand curve is downward sloping. Our particular application arises in a supply chain setting, where a manufacturer produces an item that is sold through a retailer. In this setting the manufacturer charges a fixed wholesale price in period 0. The retailer responds to this price by ordering a quantity in period

[7] For each put option that the retailer purchases in period 0, the immediate cost is $S_0 + P_0$. Now, at the end of period 1, the retailer can either return the unit of product and earn X or keep it and sell it to the market. Regardless of the choice, the retailer can always achieve exactly the same result by returning the product, and then deciding whether to repurchase it or not from the manufacturer at a price X. Each put option is therefore equivalent to a riskless income of X in period 1 plus a call option with strike X.

0. The retailer bears quantity risk, and in period 1, based on the demand curve, determines the optimal amount of inventory to release. We have shown that if the manufacturer introduces option contracts that shift some of the quantity risk away from the retailer, then the equilibrium prices adjust in a way that benefits the manufacturer and may help or harm the retailer. When volatility of the demand curve is low, then the retailer benefits from supply chain options. On the other hand, when uncertainty in the demand curve is high, the retailer may be worse off. Conditions under which the manufacturer is keen to issue option contracts were derived and shown to depend on the cost structure associated with regular production and expedited orders.

We have derived the equilibrium prices for the supply chain options. These contracts differ from financial contracts in many ways. With financial options, at expiration, arbitrage restrictions imply that there is no difference between taking delivery or cash settling the contract. For our real option, cash settlement is not a choice, and the only reason to exercise the option is to obtain an additional unit of inventory to release into the market. But because the demand curve is downward sloping, releasing an extra unit would result in lowering the retail price. Hence, exercise decisions alter retail prices, and this has a feedback effect. From the manufacturer's perspective, granting an option is only possible if expedited production is available, or if inventory building is possible. The options granted are in fixed positive supply, so they behave more like warrants in this regard. However, the pricing of options must be done simultaneously with establishing the new wholesale price. Moreover, the effects of the downward sloping curve, and its uncertainty play heavily into the pricing analysis.

Interestingly, once a call has been introduced, adding put options does not complicate the analysis. The reason is that in the presence of the call, the put has no effect on the prices of the underlying, and put-call parity must hold.

With options available, we showed that the volatility of retail prices would diminish. Further, the options do assist in coordinating the supply chain. Indeed, the channel profit equals 75% of the optimal profit that could be derived if the supply chain was managed as a single firm.

It remains for future research to extend the analysis to cases where the manufacturer's cost structure is more complex or to cases where the manufacturer offers the retailer American options that extend over multiple time periods. Other extensions include examining more complex supply chains, where a manufacturer distributes a product through a network of retailers who compete in an oligopolistic market.

Appendix

Lemma A.1 *If the manufacturer does not provide options, the optimal quantity of inventory the retailer orders at time 0 is given by:*

$$I^* = \begin{cases} \frac{a_H e_H - S_0}{2\delta e_H} & \text{if} \quad 0 < S_0 \leq (a_H - a_L)e_H \\ \frac{A_0 - S_0}{2\delta B_0} & \text{if} \quad (a_H - a_L)e_H \leq S_0 \leq A_0 \\ 0 & \text{if} \quad S_0 > A_0. \end{cases}$$

Proof

In period 1 the retailer solves the problem:

$$R_1(q^*|I, a) = Max_{0 \leq q \leq I} q(a - \delta q)$$

The optimal solution is:

$$q^* = \begin{cases} \frac{a}{2\delta} & \text{if} \quad \frac{a}{2\delta} < I \\ I & \text{if} \quad \frac{a}{2\delta} \geq I \end{cases}$$

Now consider the retailer's problem in period 0. We have:

$$R_0(I) = -IS_0 + e_L R_1(q_L^*|I) + e_H R_1(q_H^*|I).$$

There are two cases that need to be considered.

1. $0 \leq I \leq \frac{a_L}{2\delta}$
2. $\frac{a_L}{2\delta} \leq I \leq \frac{a_H}{2\delta}$

For case 1 we have

$$R_0(I_1^*) = Max_{0 \leq I \leq \frac{a_L}{2\delta}} \{-IS_0 + e_L I(a_L - \delta I) + e_H I(a_H - \delta I)\}.$$

The optimal solution is

$$I_1^* = \begin{cases} \frac{A_0 - S_0}{2\delta B_0} & \text{if} \quad (a_H - a_L)e_H \leq S_0 \leq A_0 \\ \frac{a_L}{2\delta} & \text{if} \quad S_0 < (a_H - a_L)e_H \end{cases}$$

For the second case, we have:

$$R_0(I_2^*) = Max_{\frac{a_L}{2\delta} \leq I \leq \frac{a_H}{2\delta}} \{-IS_0 + e_L \frac{a_L}{2\delta}(a_L - \delta \frac{a_L}{2\delta} + e_H I(a_H - \delta I)\}.$$

The optimal solution is

$$I_2^* = \begin{cases} \frac{a_H e_H - S_0}{2\delta e_H} & \text{if} \quad 0 < S_0 < (a_H - a_L)e_H \\ \frac{a_L}{2\delta} & \text{if} \quad S_0 \geq (a_H - a_L)e_H \end{cases}$$

The result then follows.

Proof of Proposition 1

Given the results of Lemma A.1, the manufacturer's profit as a function of S_0 is

$$M_0(S_0) = \begin{cases} M_1(S_0) & \text{if} \;\; 0 < S_0 \leq (a_H - a_L)e_H \\ M_2(S_0) & \text{if} \;\; (a_H - a_L)e_H < S_0 \leq A_0 \\ 0 & \text{if} \;\; S_0 > A_0 \end{cases},$$

where

$$M_1(S_0) = \frac{e_H a_H - S_0}{2\delta e_H}(S_0 - K_0)$$

and

$$M_2(S_0) = \frac{A_0 - S_0}{2\delta B_0}(S_0 - K_0).$$

Therefore, the maximum value of $M_0(S_0)$ is given by

$$M_0^* = M_0(S_0^*) = \max\{M_1^*, M_2^*\},$$

where

$$M_1^* = M_1(S_1^*) = Max_{0 \leq S_0 \leq (a_H - a_L)e_H} M_1(S_0) \tag{A.1}$$

and

$$M_2^* = M_2(S_2^*) = Max_{(a_H - a_L)e_H \leq S_0 \leq A_0} M_2(S_0). \tag{A.2}$$

The maximizing S_0 value for the problem in (A.1) is

$$S_1^* = \begin{cases} \frac{e_H a_H + K_0}{2} & \text{if} \quad K_0 \leq \overline{K} \\ (a_H - a_L)e_H & \text{if} \quad K_0 > \overline{K} \end{cases},$$

where the value of $\overline{K}$ is determined by solving the inequality $\frac{e_H a_H + K_0}{2} \leq (a_H - a_L)e_H$ in K_0, i.e.,

$$\overline{K} = a_H e_H - 2a_L e_H = (a_H - a_L)e_H - a_L e_H.$$

In this case the manufacturer's profits are:

$$M_1^* = \begin{cases} \frac{(a_H e_H - K_0)^2}{8\delta e_H} & \text{if} \quad K_0 \leq \overline{K} \\ \frac{[(a_H - a_L)e_H - K_0]a_L}{2\delta} & \text{if} \quad K_0 > \overline{K}. \end{cases}$$

Similarly, the maximizing S_0 value for the problem in (A.2) is

$$S_2^* = \begin{cases} \frac{A_0+K_0}{2} & \text{if} \quad K_0 \geq \underline{K} \\ (a_H - a_L)e_H & \text{if} \quad K_0 < \underline{K}, \end{cases},$$

where the value of $\underline{K}$ is determined by solving the inequality $\frac{A_0+K_0}{2} \geq (a_H - a_L)e_H$ in K_0, i.e.,

$$\underline{K} = a_H e_H - 2a_L e_H - a_L e_L = (a_H - a_L)e_H - a_L e_H(1+\rho),$$

where $\rho = e_L/e_H$.

In this case the manufacturer's profits are:

$$M_2^* = \begin{cases} \frac{(A_0-K_0)^2}{8\delta B_0} & \text{if} \quad K_0 \geq \underline{K} \\ \frac{[(a_H-a_L)e_H-K_0]a_L}{2\delta} & \text{if} \quad K_0 < \underline{K} \end{cases}$$

Since $\underline{K} < \overline{K}$, the solution can be summarized as follows

$$S_0^* = \begin{cases} \frac{a_H e_H + K_0}{2} & \text{if} \quad K_0 \leq \underline{K} \\ \frac{A_0+K_0}{2} & \text{if} \quad K_0 \geq \overline{K} \\ \frac{e_H a_H + K_0}{2} & \text{if} \quad \underline{K} < K_0 < \overline{K} \text{ and } M_1^* > M_2^* \\ \frac{A_0+K_0}{2} & \text{if} \quad \underline{K} < K_0 < \overline{K} \text{ and } M_1^* \leq M_2^* \end{cases} \tag{A.3}$$

The solution can be simplified. Substituting the values of M_1^* and M_2^* for the case where $\underline{K} < K_0 < \overline{K}$, the relationship $M_1^* - M_2^* \leq 0$ can be written as the following quadratic inequality in K_0

$$K_0^2 + 2(a_H - a_L)e_H K_0 - 2a_H a_L e_H^2 - a_L^2 e_H e_L + a_H^2 e_H^2 \leq 0.$$

The two roots of this quadratic function in K_0 are

$$\begin{aligned} k_1 &= (a_H - a_L)e_H - a_L e_H\sqrt{1+\rho} \\ k_2 &= (a_H - a_L)e_H + a_L e_H\sqrt{1+\rho} \end{aligned}$$

and the inequality is valid for $k_1 \leq K_0 \leq k_2$. In addition, in this case it must be true that $\underline{K} < K_0 < \overline{K}$. It is easy to see from the definitions of $\underline{K}$ and $\overline{K}$ that $k_2 > \overline{K}$ and $\underline{K} < k_1 < \overline{K}$. Summarizing the above relationships, it follows that, in the case where $\underline{K} < K_0 < \overline{K}$, $M_1^* - M_2^* \leq 0$ is true if and only if $k_1 < K_0 < \overline{K}$. Based on this, equation (A.3) can simplified as follows.

$$S_0^* = \begin{cases} \frac{a_H e_H - K_0}{2} & \text{if} \quad K_0 \leq k_1 \\ \frac{A_0+K_0}{2} & \text{if} \quad K_0 > k_1 \end{cases} \tag{A.4}$$

Using the transformations (11) - (14), we find

$$\frac{a_H e_H - K_0}{2} = S_{det} - \frac{e_L}{2}(\frac{A_0}{B_0} - \frac{\sigma_A}{\sqrt{\rho}}).$$

Further, the condition $K_0 \leq k_1$ can be reexpressed as an equivalent condition involving the volatility of the demand curve. In particular, substituting $a_H = \frac{A_0}{B_0} + \sqrt{\rho}\sigma_A$ and $a_L = \frac{A_0}{B_0} - \frac{\sigma_A}{\sqrt{\rho}}$ into the expression for k_1, we find that $K_0 \leq k_1$ if and only if

$$\sigma_A \geq \eta \equiv \frac{\sqrt{\rho}}{1+\rho+\sqrt{1+\rho}}\left[(1+\rho)\frac{K_0}{B_0} + \sqrt{1+\rho}\frac{A_0}{B_0}\right].$$

Therefore equation (15) for the optimal wholesale price is established.

To complete the proof, it remains to determine the retailer's policy in each of the two cases.

In the case when $K_0 \leq k_1$, we have that $S_0^* = \frac{a_H e_H + K_0}{2} < (a_H - a_L)e_H$, and the results of the above lemma imply that

$$I^* = \frac{a_H e_H - S_0^*}{2\delta e_H} = \frac{a_H e_H - K_0}{4\delta e_H}.$$

Furthermore, it is easy to see that $I^* < \frac{a_H}{2\delta}$, therefore, $q_H^* = I^*$. On the other hand,

$$I^* - \frac{a_L}{2\delta} = \frac{a_H e_H - 2a_L e_H - K_0}{4\delta e_H} = \frac{\overline{K} - K_0}{4\delta e_H} > 0,$$

since $K_0 \leq k_1 < \overline{K}$. Thus, $I^* > \frac{a_L}{2\delta}$ and $q_L^* = \frac{a_L}{2\delta}$.

Finally, in the case when $K_0 > k_1$, it follows that

$$(a_H - a_L)e_H < S_0^* = \frac{A_0 + K_0}{2} < A_0,$$

and the results of the above lemma imply that

$$I^* = \frac{A_0 - S_0^*}{2\delta B_0} = \frac{A_0 - K_0}{4\delta B_0}.$$

Furthermore,

$$I^* - \frac{a_L}{2\delta} = \frac{A_0 - 2a_L B_0 - K_0}{4\delta B_0} == \frac{a_H e_H - 2a_L e_H - a_L e_L}{4\delta B_0} = \frac{\underline{K} - K_0}{4\delta B_0} < 0,$$

since $K_0 > k_1 > \underline{K}$. Hence $I^* < \frac{a_L}{2\delta} < \frac{a_H}{2\delta}$, and $q_L^* = q_H^* = I^*$. Equations (17), (18), (19) follow from the substitutions (11) to (14). This completes the proof.

Proof of Lemma 1

Let

$$\begin{aligned} R_1^{(1)}(q) &= R_1(q,0) = q(q - \delta q) \\ R_1^{(2)}(v) &= R_1(I,v) = (I+v)(a - \delta(I+v)) - Xv. \end{aligned}$$

Then, since no options will be exercised when inventory is available, we have:

$$\max_{0 \leq q \leq I, 0 \leq v \leq U} R_1(q,v) = \max\{\max_{0 \leq q \leq I} R_1^{(1)}(q), \max_{0 \leq v \leq U} R_1^{(2)}(v)\}.$$

We also have

$$\begin{aligned}
\frac{dR_1^{(1)}}{dq}(I) &= a - 2\delta I \\
\frac{dR_1^{(2)}}{dv}(0) &= a - 2\delta I - X
\end{aligned}$$

First, assume $\frac{dR_1^{(1)}}{dq}(I) < 0$. This implies that $R_1^{(1)}(q)$ is maximized for $q = q^* = \frac{a}{2\delta} < I$. Further, for this case, $\frac{dR_1^{(2)}}{dv}(0) < 0$. This and the concavity of $R_1^{(2)}(v)$ imply that $\frac{dR_1^{(2)}}{dv}(v) < 0$ for all $v > 0$. Hence $Max_{0 \le v \le U}[R_1^{(2)}(v)] = R_1^{(2)}(0)$. This implies that:

$$R_1^{(2)}(v) \le R_1^{(2)}(0) = R_1(I, 0) = R^{(1)}(I) \le R_1^{(1)}(q^*)$$

which means that $q = q^*, v = 0$ is optimal in this case.

Second, consider the case where $I < \frac{a}{2\delta}$. In this case, $\frac{dR_1^{(1)}}{dq}(I) > 0$. Hence:

$$R_1^{(1)}(q) \le R_1^{(1)}(I) = R_1^{(2)}(0) \le R_1^{(2)}(v^*)$$

where v^* is the value maximizing $R_1^{(2)}(v)$. Specifically, using the first order conditions for $R_1^{(2)}(v)$ we obtain:

$$v^* = \begin{cases} 0 & \text{if} \quad I + U > \frac{a-X}{2\delta} \\ \frac{a-X}{2\delta} - I & \text{if} \quad \frac{a-X}{2\delta} > I \end{cases}$$

Proof of Lemma 2

First, consider the regions

1. $I < \frac{a_L - X}{2\delta}$ and $U > \frac{a_H - X}{2\delta} - I$.
2. $\frac{a_L - X}{2\delta} < I < \frac{a_H - X}{2\delta}$ and $U > \frac{a_H - X}{2\delta} - I$.
3. $I > \frac{a_H - X}{2\delta}$ and $U > 0$.

In each of these regions, regardless of which state occurs in the future, the maximum number of options that can be exercised, U, is never attained. Hence, if $C_0 > 0$, then clearly U can be reduced and the retailer can obtain savings. Hence the optimal solution for the retailer will never lie in these regions.

Second, we show that if $U > 0$ and $I < \frac{(a_L - X)}{2\delta}$, then (I, U) cannot be optimal.

If $X > a_L$, then $a_L - X < 0$ and $I > (a_L - X)/2\delta$. Now consider $X \le a_L$. Take (I, U) such that $U > 0$ and $I \le \frac{a_L - X}{2\delta}$. Then $v_L^*, v_H^* > 0$, for all $U > 0$. Now

$$R_0(I, U) = -S_0 I - C_0 U + e_L(I + v_L^*)(a_L - \delta(I + v_L^*) - X e_L v_L^* + e_H(I + v_H^*)(a_H - \delta(I + v_H^*) - X e_H v_H^*$$

Let $I' = I + \epsilon$, $U' = U - \epsilon$, where $0 < \epsilon < Min[v_L^*, v_H^*]$. Then $v_L' = v_L^* - \epsilon < U - \epsilon = U'$ and $v_H' = v_H^* - \epsilon < U - \epsilon = U'$ are feasible, but perhaps not optimal, exercise policies. Also, $I' + v_L' = I + v_L^*$, and $I' + v_H' = I + v_H^*$. Then:

$$R_0(I', U') \geq -S_0 I' - C_0 U' + e_L(I+v_L^*)(a_L - \delta(I+v_L^*)) - Xe_L(v_L^* - \epsilon) + e_H(I+v_H^*)(a_H - \delta(I+v_H^*)) - Xe_H(v_H^* - \epsilon)$$

Hence, $R_0(I', U') - R_0(I, U) \geq (C_0 - S_0 + XB_0)\epsilon > 0$. Therefore, (I, U) is not optimal.

The only region that remains when $U > 0$ is the region where $I > \frac{a_L - X}{2\delta}$ and $U \leq \frac{a_H - X}{2\delta} - I$. This completes the proof.

Proof of Proposition 5

(i) First, consider the case where $\sigma_A^2 < \eta^2$ and $\widetilde{K_L} \leq K_1 \leq \widetilde{K_H}$. Then, computing the difference between the manufacturer's profit with and without options, leads, upon simplification to:

$$M^* - M_0 = \frac{e_H}{8\delta e_L B_0}[a_L e_L - a_H e_L - K_0 + e_H K_1 + K_1 e_L]^2 > 0$$

which shows that, for this case, the manufacturer is better off with options.

(ii) Now consider the case where $\sigma_A^2 \geq \eta^2$ and $\widetilde{K_L} \leq K_1 \leq \widetilde{K_H}$. Then, computing the difference between the manufacturer's profit with and without options, leads, upon simplification to:

$$M^* - M_0 = \frac{N(K_1)}{8\delta e_H e_L}$$

where

$$N(K_1) = a_0 K_1^2 + b_0 K_1 + c_0$$

and

$$\begin{aligned} a_0 &= e_H^2 B_0 \\ b_0 &= -2e_H^2[K_0 + (a_H - a_L)e_L] \\ c_0 &= (e_H - e_L)K_0^2 + e_H e_L^2 a_L^2 + 2K_0 e_H e_L (a_H - a_L) \end{aligned}$$

Recall that the volatility range in (32) is equivalent to the range of costs in (31). Furthermore, since in the case we examine $\sigma_A > \eta$ and from (16) $\eta \geq \sqrt{\rho}\frac{K_0}{B_0}$, it follows that $\sigma > \sqrt{\rho}\frac{K_0}{B_0}$ and the upper bound in (31) becomes $\bar{k}_1(\sigma_A) = \frac{K_0}{e_H}$.

In addition,

$$N(\bar{k}_1) = e_H e_L^2 a_L^2$$

and

$$N'(\bar{k}_1) = 2e_H e_L \left[K0 - (a_H - a_L)e_H\right],$$

which, using the substitutions (11)-(14), becomes

$$N'(\bar{k}_1) = 2\frac{B_0^2\sqrt{\rho}\left(\sqrt{\rho}K_0 - \sigma B_0\right)}{(1+\rho)^2}$$

As we have seen above, $\sigma > \sqrt{\rho}\frac{K_0}{B_0}$, thus $N'(\bar{k}_1) < 0$. Therefore, $N(K_1)$ is a convex quadratic function which is positive and decreasing at the upper bound of the range of K_1. It is thus positive in the entire range $\underline{k}_1(\sigma_A) \leq K_1 \leq \overline{k}_1(\sigma_A)$, for any $\sigma_A > \eta$. This completes the proof of the proposition.

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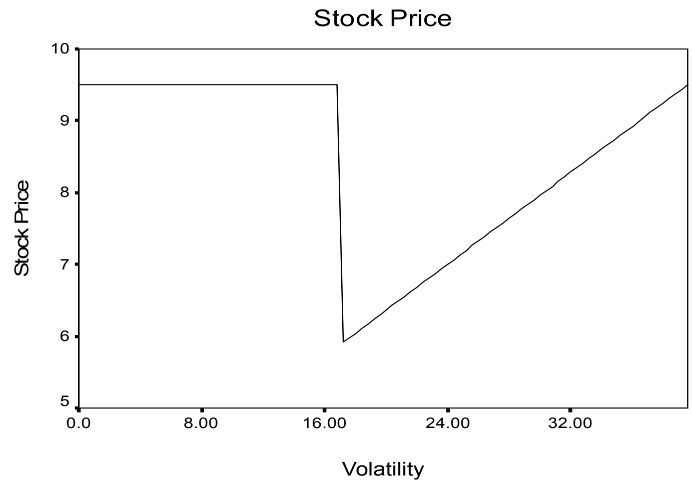
Figure 1:
Sensitivity to Volatility for the No Options Case

The figures shows the equilibrium wholesale price, inventory levels, and profits of the manufacturer and retailer as volatility of the demand curve increases from its minimum value to its maximum. In this case no options are allowed. The case parameters are:

$$B_0 = 0.8, A_0 = 16, \rho = 4, \delta = 1, K_0 = 3.$$



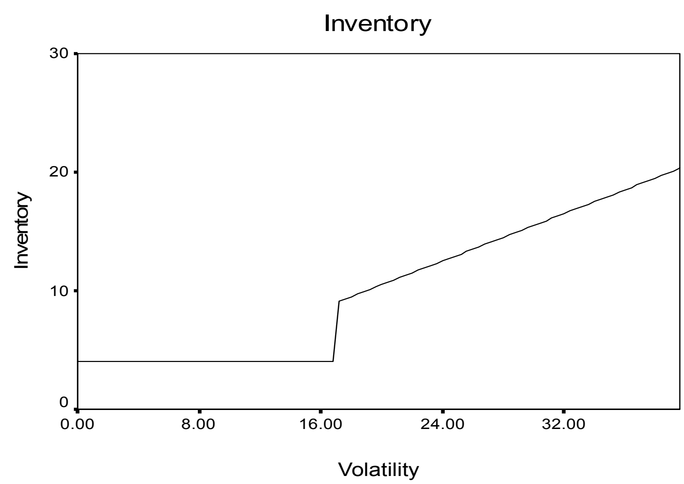
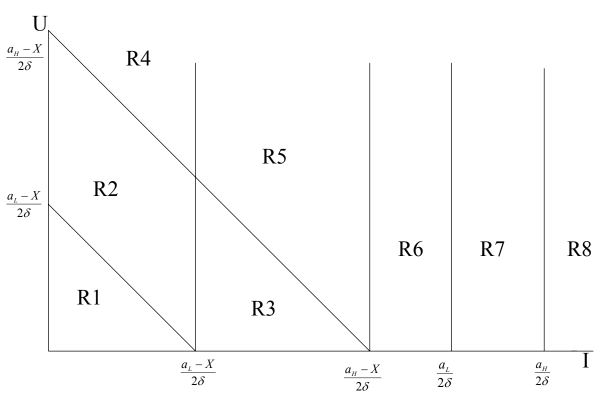


Figure 2
Retailer's Optimal Policy Regions in Period 1

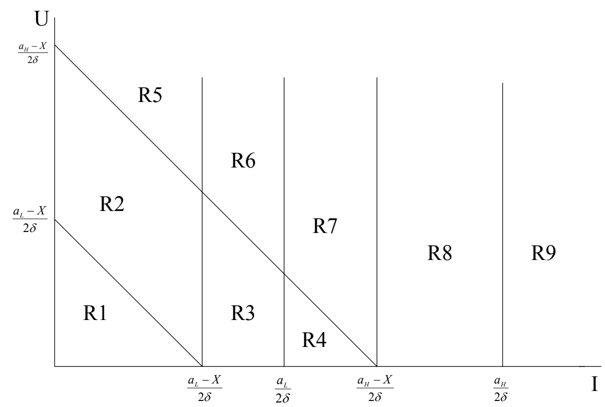
The figure shows the optimal policies for the retailer in period 1, conditional on inventory and option decicions taken in period 0, for each of the two realizations of the demand curve. For example, if [I,U] fall in the region R2, and the demand realization is high, then, for both cases, the optimal policy is for the retailer to release all inventory and exercise all options.

Case 1: $a_H - a_L < X$



Region	q_L^*	v_L^*	q_H^*	v_H^*
R1	I	U	I	U
R2	I	$\frac{a_L - X}{2\delta} - I$	I	U
R3	I	0	I	U
R4	I	$\frac{a_L - X}{2\delta} - I$	I	$\frac{a_H - X}{2\delta} - I$
R5	I	0	I	$\frac{a_H - X}{2\delta} - I$
R6	I	0	I	0
R7	$\frac{a_L}{2\delta}$	0	I	0
R8	$\frac{a_L}{2\delta}$	0	$\frac{a_H}{2\delta}$	0

Case 2: $a_H - a_L \geq X$



Region	q_L^*	v_L^*	q_H^*	v_H^*
R1	I	U	I	U
R2	I	$\frac{a_L - X}{2\delta} - I$	I	U
R3	I	0	I	U
R4	$\frac{a_L}{2\delta}$	0	I	$\frac{a_H - X}{2\delta} - I$
R5	I	$\frac{a_L - X}{2\delta} - I$	I	$\frac{a_H - X}{2\delta} - I$
R6	I	0	I	$\frac{a_H - X}{2\delta} - I$
R7	$\frac{a_L}{2\delta}$	0	I	0
R8	$\frac{a_L}{2\delta}$	0	$\frac{a_H}{2\delta}$	0

Figure 3:
Sensitivity to Volatility for the Options Case

The figures shows the equilibrium wholesale and option prices; inventory and option quantities; and profits of the manufacturer and retailer as volatility of the demand curve increases from its minimum value to its maximum. The case parameters are:

$$B_0 = 0.8, A_0 = 16, \rho = 4, \delta = 1, K_0 = 3, K_1 = 10, X = 12.$$

For this value of K_1 the valid range of σ for options to be used is (3.125, 35.625).

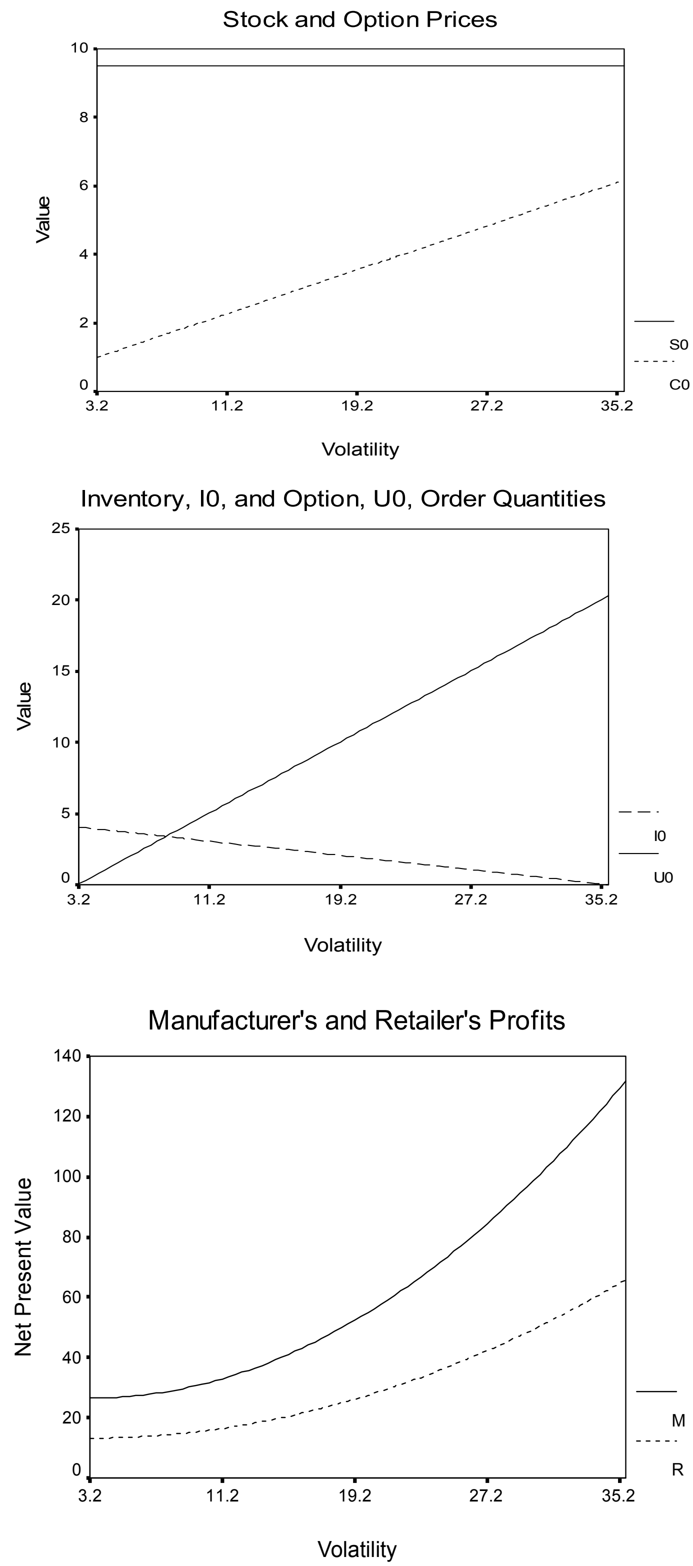
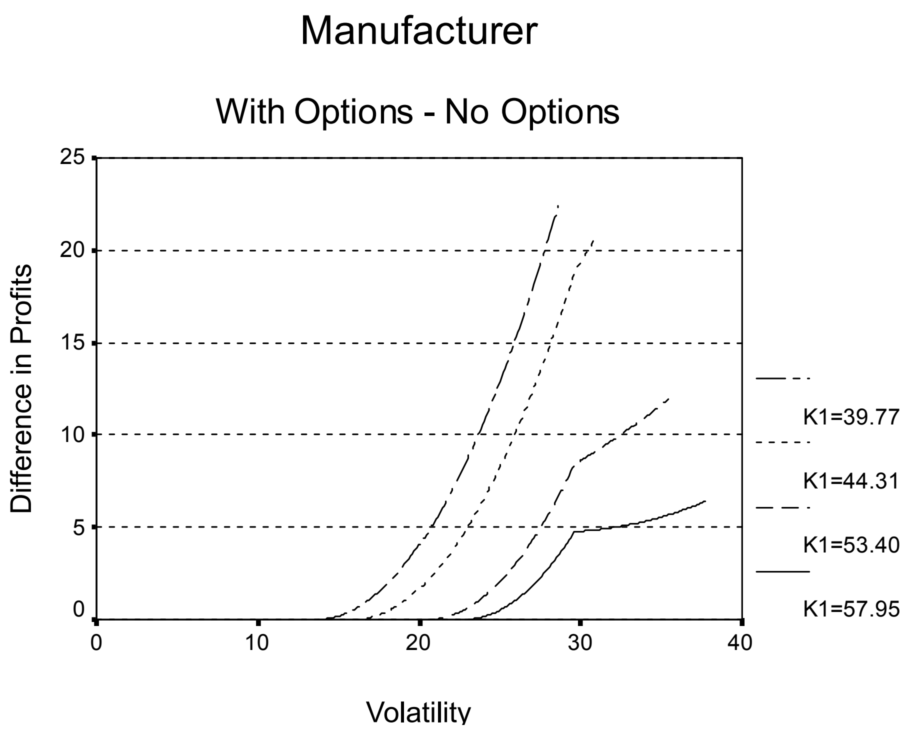


Figure 4
The Value of the Option for the Two Parties

The figures show the value of the option for different values of the volatility of the demand curve. The top figure shows the value of the option for the manufacturer for every feasible value of the volatility, for 4 different per unit costs of manufacturing in period 1. The bottom figure shows the value of the option for the retailer for the same four manufacturing costs. The value of the option is defined to be the optimal profit with options less the optimal profit if the manufacturer does not consider using options. The case parameters are as in Figure 3.



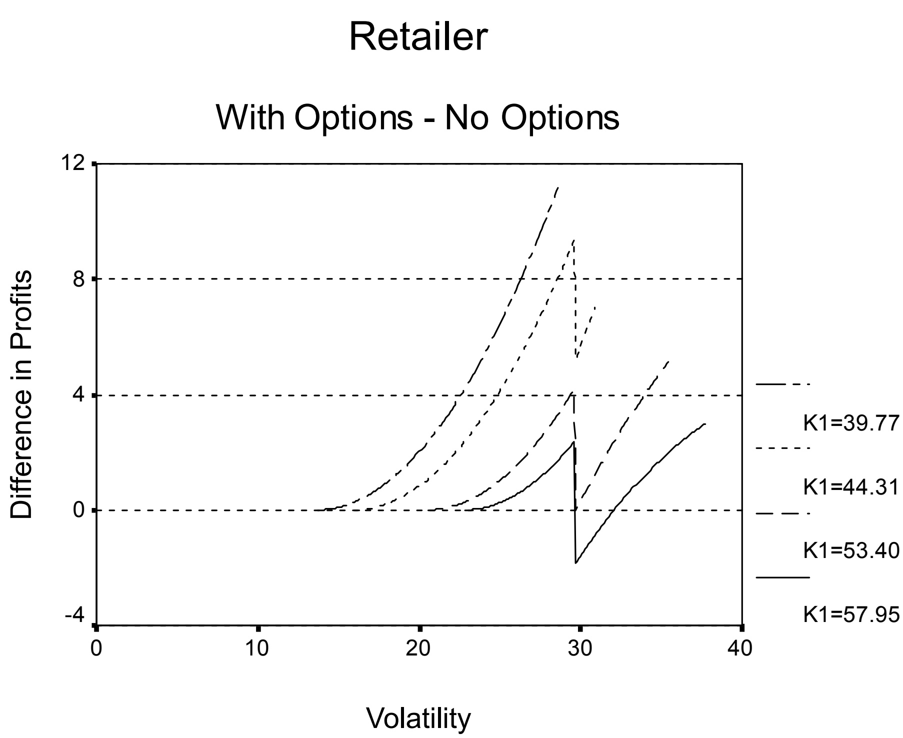


Figure 5
Comparison of Channel Profits

The top panel compares the values of the channel with no options and with options, against the optimal coordinated channel value. For the case where options are used, and for the case where there is a coordinated (single) channel, we require the costs of expediting an order in period 1. Two cases are taken. For case (1) K_1=35.25. For case (2) K_1=57.95. All the remaining parameter values are as before.
The second panel shows how options improve the channel profits. In particular, the percentage improvements over the no-option case is reported over the range of volatilities. Also shown is the maximum possible improvement that could be obtained under perfect coordination (a single owner).
The third figure shows the fraction of the total possible improvement that could be obtained (under one ownership) which is achieved by introducing options.