5

Eurodollar Futures, and Forwards

In this chapter we will learn about

- Eurodollar Deposits
- Eurodollar Futures Contracts,
- Hedging strategies using ED Futures,
- Forward Rate Agreements,
- Pricing FRAs.
- Hedging FRAs using ED Futures,
- Constructing the Libor Zero Curve from ED deposit rates and ED Futures.

5.1 EURODOLLAR DEPOSITS

As discussed in chapter 2, Eurodollar (ED) deposits are dollar deposits maintained outside the USA. They are exempt from Federal Reserve regulations that apply to domestic deposit markets. The interest rate that applies to ED deposits in interbank transactions is the LIBOR rate. The LIBOR spot market has maturities from a few days to 10 years but liquidity is the greatest

Table 5.1 LIBOR spot rates

Dates	7day	1mth.	3mth	6mth	9mth	1yr
LIBOR	1.000	1.100	1.160	1.165	1.205	1.337

within one year. Table 5.1 shows LIBOR spot rates over a year as of January 14^{th} 2004.

In the ED deposit market, deposits are traded between banks for ranges of maturities. If one million dollars is borrowed for 45 days at a LIBOR rate of 5.25%, the interest is

$$\text{Interest } = 1m \times 0.0525 \frac{45}{360} = \$6562.50$$

The rate quoted assumes settlement will occur two days after the trade.

Banks are willing to lend money to firms at the Libor rate provided their credit is comparable to these strong banks. If their credit is weaker, then the lending bank may quote a rate as a spread over the Libor rate.

5.2 THE TED SPREAD

Banks that offer LIBOR deposits have the potential to default. The 3 month LIBOR rate will therefore be set higher than the 3-month Treasury rate, with the spread between the two rates representing a premium for default. This Treasury-Eurodollar spread, called the TED spread fluctuates with time. When the economy is sound, spreads may be fairly small, but in times of crises, the spreads can be fairly large. The range in spreads can typically be between 30 to 300 basis points.

A firm that borrows funds over time will typically face interest rates more highly correlated with LIBOR than with Treasury rates. Hence, rather than use futures contracts on Treasuries as a hedge, firms typically will look to hedge short term rates using Eurodollar (ED) futures.

5.3 90 DAY EURODOLLAR FUTURES

The 90 day LIBOR rate is the yield derived on a 90 day ED deposit. ED futures contracts that settle to a 90 day LIBOR rate are very actively traded.[1]

The underlying security is a $\$1,000,000$ 90-day Libor deposit. The futures contracts available mature in March, June, September and December of each year and extend out for about 10 years. In addition, the four nearest contract months also trade. The ED futures contract settles by cash on its expiration date, which is the second London business day before the third Wednesday of the maturity month. On the expiration date, the final settlement price is determined by the clearing house using the following procedure. At the close of trading, a sample of 12 banks from a list of no more than 20 participating banks is randomly selected. These banks provide a quotation on 3 month LIBOR deposit rates. The clearing house eliminates the lowest 2 and highest two quotes, and takes the arithmetic average of the remaining 12 quotes. This procedure is repeated at a random time within 90 minutes. The average of these two prices determines the final LIBOR rate. The final settlement price is established by subtracting this rate from 100. The average is rounded to the nearest 1/10000th of a percentage point, with decimal fractions ending in a five rounded up. For example, an average rate of 8.65625% would be rounded to 8.6563 and then subtracted from 100 to determine a final settlement price of 91.3437.

Let t_0 be the expiration date of the futures contract, and let $\ell[t_0, t_1]$ be the LIBOR rate, expressed in annualized decimal form, at date t_0. Since the settlement date is two days later, the rate $\ell[t_0, t_1]$ actually represents the rate from date $t_0 + 2$ and t_1 would be 90 days later. The final quoted futures *index* price is

$$FU(t_0) = 100[1 - \ell[t_0, t_1]].$$

Let $FU(t)$ be the quoted index futures price at an earlier date t, $t < t_0$. The implied LIBOR interest rate is $[100 - FU(t)]\%$, or in decimal form:

$$I\ell_t[t_0, t_1] = 1 - \frac{FU(t)}{100}$$

Thus, a futures index of 92 corresponds to an implied interest rate of 8%, or $I\ell_t[t_0, t_1] = 0.08$.

The actual or effective dollar price of the contract differs from the index because the yield is divided by four, so as to reflect a three month rate.

$$\text{Effective Price } = 1m \times (1 - \frac{I\ell_t[t_0, t_1]}{4})$$

[1]The futures contract traded on the CME is one of the most actively traded futures contracts in the world.

Example

A trader buys a Eurodollar futures price at 92.0 and sells it the same day at 92.08. The change of 8 basis points causes a price change of \$200. Specifically, the profit to the long is given by:

$$\begin{aligned} & 1m \times (1 - \frac{0.0792}{4}) - 1m \times (1 - \frac{0.080}{4}) \\ = \quad & 1m \times (\frac{0.0800 - 0.0792}{4}) \\ = \quad & 1m \times 0.0002 = \$200. \end{aligned}$$

Thus each basis point change in the annualized implied LIBOR rate is worth \$25.

A long position in a ED futures contract is really just a short position on the 90 day LIBOR interest rate. In the last example the implied LIBOR rate decreased by 8 basis points, from 8.0% to 7.92%, leading to a profit on the long position of $8 \times 25 = \$200$. A trader who sells ED futures, profits if rates increase.

ED futures contracts that trade on SIMEX use the CME's final settlement price. These two exchanges designed a system that permits futures contracts traded on SIMEX to be completely interchangeable with contracts that trade on the CME. The contract specifications on both exchanges are identical except for trading hours. However, a contract that trades on the floor of the CME can be transferred through the *mutual offset system* to SIMEX, and cancelled there. Similarly, a contract traded on the floor of SIMEX can be transferred to the CME. As a result, the trading hours of these contracts extends beyond the trading hours of either exchange.

Table (5.2) shows the ED futures contracts that traded on January 15th 2004. The cash settlement date for each contract is indicated as well as the gap, measured in days, between successive expiration dates. The settlement price is indicated, and information from the previous day is supplied.

Notice that actively traded ED Futures contracts exist with settlement dates that extend beyond 5 years. This is quite unusual. In most futures markets, liquidity drops off very rapidly with maturity. The reason for high liquidity even in distant contracts relates to the use of these contracts as hedges for FRAs, Interest Rate Swaps, and other derivatives. This will be discussed later.

As an example, consider the June 2005 contract. The current implied Libor rate for this contract is $100 - 97.315 = 2.685\%$. A speculator who thinks that the 90 day spot Libor rate beginning from the June 2005 expiration date will be much higher than this number might consider selling this futures contract.

Table 5.2 Eurodollar Futures Information: January 15th 2004

Seq. No.	Contract Month	Product Code	Cash Settlement Date	Days to next contract	Settle Price	Prior Day			Implied Futures	Discount Factor for Period	Discount Factor from Date 0
						Settle	Vol	Open Int.			
3	Mar-04	EDH4	3/15/04	91	98.84	98.84	74123	816783	1.16	0.99807	0.99807
6	Jun-04	EDM4	6/14/04	91	98.72	98.74	115766	778181	1.28	0.99708	0.99515
7	Sep-04	EDU4	9/13/04	91	98.47	98.5	176217	699425	1.53	0.99677	0.99194
8	Dec-04	EDZ4	12/13/04	91	98.11	98.155	214525	594703	1.89	0.99615	0.98812
9	Mar-05	EDH5	3/14/05	91	97.715	97.775	129631	398793	2.285	0.99525	0.98342
10	Jun-05	EDM5	6/13/05	98	97.315	97.38	50644	304758	2.685	0.99426	0.97778
11	Sep-05	EDU5	9/19/05	91	96.975	97.035	40731	239541	3.025	0.99274	0.97068
12	Dec-05	EDZ5	12/19/05	84	96.69	96.74	37560	175919	3.31	0.99241	0.96331
13	Mar-06	EDH6	3/13/06	98	96.465	96.505	16530	149915	3.535	0.99234	0.95593
14	Jun-06	EDM6	6/19/06	91	96.255	96.285	9878	125749	3.745	0.99047	0.94682
15	Sep-06	EDU6	9/18/06	91	96.055	96.075	13718	112197	3.945	0.99062	0.93794
16	Dec-06	EDZ6	12/18/06	91	95.86	95.87	12413	99096	4.14	0.99013	0.92868
17	Mar-07	EDH7	3/19/07	91	95.7	95.7	5012	73199	4.3	0.98964	0.91906
18	Jun-07	EDM7	6/18/07	91	95.54	95.53	4857	64220	4.46	0.98925	0.90918
19	Sep-07	EDU7	9/17/07	91	95.395	95.375	5750	71185	4.605	0.98885	0.89904
20	Dec-07	EDZ7	12/17/07	91	95.245	95.215	6965	57673	4.755	0.98849	0.88870
21	Mar-08	EDH8	3/17/08	91	95.13	95.095	5347	46368	4.87	0.98812	0.87814
22	Jun-08	EDM8	6/16/08	91	95.015	94.975	5246	48745	4.985	0.98784	0.86747
23	Sep-08	EDU8	9/15/08	91	94.91	94.865	5106	34643	5.09	0.98756	0.85667
24	Dec-08	EDZ8	12/15/08	91	94.795	94.75	5798	25128	5.205	0.98730	0.84579
25	Mar-09	EDH9	3/16/09	91	94.705	94.655	224	10810	5.295	0.98701	0.83481
26	Jun-09	EDM9	6/15/09	91	94.635	94.585	186	9417	5.365	0.98679	0.82378
27	Sep-09	EDU9	9/14/09	91	94.57	94.52	183	8415	5.43	0.98662	0.81276
28	Dec-09	EDZ9	12/14/09	91	94.485	94.435	183	5244	5.515	0.98646	0.80175
29	Mar-10	EDH0	3/15/10	91	94.41	94.36	26	8001	5.59	0.98625	0.79073
30	Jun-10	EDM0	6/14/10	91	94.345	94.295	25	6757	5.655	0.98607	0.77971
31	Sep-10	EDU0	9/13/10	91	94.275	94.225	25	4682	5.725	0.98591	0.76872
32	Dec-10	EDZ0	12/13/10	91	94.21	94.16	25	2696	5.79	0.98573	0.75776
33	Mar-11	EDH1	3/14/11	91	94.145	94.095	15	3183	5.855	0.98558	0.74683
34	Jun-11	EDM1	6/13/11	98	94.07	94.02	15	1448	5.93	0.98542	0.73593
35	Sep-11	EDU1	9/19/11	91	94.01	93.96	15	1960	5.99	0.98411	0.72424
36	Dec-11	EDZ1	12/19/11	91	93.94	93.89	15	1556	6.06	0.98508	0.71344
37	Mar-12	EDH2	3/19/12	91	93.87	93.82	25	1374	6.13	0.98491	0.70268
38	Jun-12	EDM2	6/18/12	91	93.795	93.745	25	1059	6.205	0.98474	0.69195
39	Sep-12	EDU2	9/17/12	91	93.735	93.685	25	728	6.265	0.98456	0.68127
40	Dec-12	EDZ2	12/17/12	91	93.665	93.615	26	238	6.335	0.98441	0.67065
41	Mar-13	EDH3	3/18/13	91	93.59	93.54	15	211	6.41	0.98424	0.66008
42	Jun-13	EDM3	6/17/13	91	93.515	93.465	15	359	6.485	0.98406	0.64955
43	Sep-13	EDU3	9/16/13	91	93.445	93.395	15	148	6.555	0.98387	0.63908
44	Dec-13	EDZ3	12/16/13	-	93.34	93.29	15	70	6.66	0.98370	0.62866

Total Volume	942969
Open Interest	5137277

A plot of these successive 90 day implied futures rates against time, produces a curve that, in this case, rises from 1.125% to 6.66%. If we incorrectly assumed that these futures rates were equal to forward rates, then we could construct a LIBOR yield curve or a LIBOR discount bond curve.

5.4 CONSTRUCTING THE LIBOR DISCOUNT FUNCTION

In Table (5.2), the first ED futures contract expires in 60 days. Assume the 60-day spot LIBOR rate was 1.16%. Then, if $\frac{1}{(1+0.016\times60/360)} = 0.99807$ dollars were placed in a ED deposit, after 60 days this would be worth \$1.0. This discount factor is recorded in the second last column of the table. By selling the March futures contract, the trader locks into a rate of 1.160% for the period from March to June. The implied discount factor for that period is therefore $\frac{1}{(1+0.016\times91/360)} = 0.99708$. The discount factor from January 15th 2004 to 14th June 2004 is therefore the product of this number with the previous discount factor. This value, 0.99515 is indicated in the second row of the last column.

In general, then, given the discount factor up to date t_j is $P(0,t_j)$, and given the implied ED futures rate for the next period $[t_j, t_{j+1}]$, we compute the discount factor up to date t_{j+1} as follows:

$$P(0,t_{j+1}) = P(0,t_j)\frac{1}{(1 + I\ell[t_j, t_{j+1}] \times (t_{j+1} - t_j)/360)}$$

The resulting plot of these values against maturity constitute the LIBOR discount function.

The above analysis would be precise is the implied futures rates were forward rates. Recall that forward prices (rates) are NOT equal to futures prices (rates) when interest rates are uncertain. The computation of forward rates from these futures rates requires an adjustment downward.

To see that a downward adjustment is necessary, recall that futures contracts are resettled daily. This resetlement process implies that there are daily cash flows into and out of the ED futures account. Consider a long position in the futures contract. When rates drop, the futures price increases, producing cash inflows. Unfortunately, these profits are reinvested at the lower rate. Conversely, when rates rise, the long position will lose money, and these losses have to be financed at higher rates. The negative impact of these reinvestments and borrowings must be compensated by a lower initial contract price, relative to the more advantageous forward contract. Lower ED futures prices imply higher initial ED implied rates.

The magnitude of the adjustment, referred to as the *convexity adjustment*, can be quantified and is the topic of a future chapter. The magnitude of the adjustment depends on the volatility of spot LIBOR and on the maturity of the futures contract. A very good approximation to the adjustment is given by the formula:

$$Adj = 10,000 \times \sigma^2(\frac{T^2}{2} + \frac{T}{8}).$$

This adjustment is in basis points. The volatilty, σ is typically less than 0.01. The maturity, T, is measured in years from the current date to the expiration date of the futures. Using this value the downward adjustment to the implied futures rate for all our ED futures contracts are shown in Table (5.3).

Table 5.3 Eurodollar Futures Information: January 15th 2004

Seq. No.	Contract Month	Product Code	Cash Settlement Date	Days to next contract	Time to Maturity (years)	Settle Price	Implied Futures	Adjustment Factor (basis points)	Estimated Forward Rate	Discount Factor for Period	Discount Factor from Date 0
3	Mar-04	EDH4	3/15/04	91	0.16393	98.84	1.16	0.034	1.1597	0.99807	0.99807
6	Jun-04	EDM4	6/14/04	91	0.41257	98.72	1.28	0.137	1.2786	0.99708	0.99515
7	Sep-04	EDU4	9/13/04	91	0.66120	98.47	1.53	0.301	1.5270	0.99678	0.99195
8	Dec-04	EDZ4	12/13/04	91	0.90984	98.11	1.89	0.528	1.8847	0.99615	0.98813
9	Mar-05	EDH5	3/14/05	91	1.16005	97.715	2.285	0.818	2.2768	0.99526	0.98345
10	Jun-05	EDM5	6/13/05	98	1.40903	97.315	2.685	1.169	2.6733	0.99428	0.97782
11	Sep-05	EDU5	9/19/05	91	1.67715	96.975	3.025	1.616	3.0088	0.99278	0.97076
12	Dec-05	EDZ5	12/19/05	84	1.92613	96.69	3.31	2.096	3.2890	0.99245	0.96343
13	Mar-06	EDH6	3/13/06	98	2.15693	96.465	3.535	2.596	3.5090	0.99238	0.95609
14	Jun-06	EDM6	6/19/06	91	2.42518	96.255	3.745	3.244	3.7126	0.99054	0.94704
15	Sep-06	EDU6	9/18/06	91	2.67427	96.055	3.945	3.910	3.9059	0.99070	0.93824
16	Dec-06	EDZ6	12/18/06	91	2.92336	95.86	4.14	4.638	4.0936	0.99022	0.92907
17	Mar-07	EDH7	3/19/07	91	3.17317	95.7	4.3	5.431	4.2457	0.98976	0.91955
18	Jun-07	EDM7	6/18/07	91	3.42231	95.54	4.46	6.284	4.3972	0.98938	0.90979
19	Sep-07	EDU7	9/17/07	91	3.67146	95.395	4.605	7.199	4.5330	0.98901	0.89979
20	Dec-07	EDZ7	12/17/07	91	3.92060	95.245	4.755	8.176	4.6732	0.98867	0.88959
21	Mar-08	EDH8	3/17/08	91	4.16804	95.13	4.87	9.207	4.7779	0.98832	0.87921
22	Jun-08	EDM8	6/16/08	91	4.41708	95.015	4.985	10.307	4.8819	0.98807	0.86871
23	Sep-08	EDU8	9/15/08	91	4.66612	94.91	5.09	11.470	4.9753	0.98781	0.85812
24	Dec-08	EDZ8	12/15/08	91	4.91516	94.795	5.205	12.694	5.0781	0.98758	0.84747
25	Mar-09	EDH9	3/16/09	91	5.16515	94.705	5.295	13.985	5.1551	0.98733	0.83673
26	Jun-09	EDM9	6/15/09	91	5.41423	94.635	5.365	15.334	5.2117	0.98714	0.82596
27	Sep-09	EDU9	9/14/09	91	5.66332	94.57	5.43	16.745	5.2626	0.98700	0.81522
28	Dec-09	EDZ9	12/14/09	91	5.91241	94.485	5.515	18.217	5.3328	0.98687	0.80452
29	Mar-10	EDH0	3/15/10	91	6.16230	94.41	5.59	19.757	5.3924	0.98670	0.79382
30	Jun-10	EDM0	6/14/10	91	6.41142	94.345	5.655	21.355	5.4415	0.98655	0.78315
31	Sep-10	EDU0	9/13/10	91	6.66054	94.275	5.725	23.014	5.4949	0.98643	0.77252
32	Dec-10	EDZ0	12/13/10	91	6.90966	94.21	5.79	24.735	5.5426	0.98630	0.76194
33	Mar-11	EDH1	3/14/11	91	7.15948	94.145	5.855	26.524	5.5898	0.98618	0.75141
34	Jun-11	EDM1	6/13/11	98	7.40862	94.07	5.93	28.370	5.6463	0.98607	0.74094
35	Sep-11	EDU1	9/19/11	91	7.67693	94.01	5.99	30.427	5.6857	0.98486	0.72972
36	Dec-11	EDZ1	12/19/11	91	7.92608	93.94	6.06	32.402	5.7360	0.98583	0.71938
37	Mar-12	EDH2	3/19/12	91	8.17336	93.87	6.13	34.424	5.7858	0.98571	0.70910
38	Jun-12	EDM2	6/18/12	91	8.42245	93.795	6.205	36.522	5.8398	0.98559	0.69888
39	Sep-12	EDU2	9/17/12	91	8.67153	93.735	6.265	38.682	5.8782	0.98545	0.68871
40	Dec-12	EDZ2	12/17/12	91	8.92062	93.665	6.335	40.904	5.9260	0.98536	0.67863
41	Mar-13	EDH3	3/18/13	91	9.17054	93.59	6.41	43.196	5.9780	0.98524	0.66862
42	Jun-13	EDM3	6/17/13	91	9.41966	93.515	6.485	45.542	6.0296	0.98511	0.65866
43	Sep-13	EDU3	9/16/13	91	9.66877	93.445	6.555	47.951	6.0755	0.98499	0.64877
44	Dec-13	EDZ3	12/16/13	-	9.91788	93.34	6.66	50.422	6.1558	0.98487	0.63896

If the maturity is less than one year, the magnitude of the difference is usually less than a basis point. However, for longer dated contracts the convexity affect can be fairly significant. For example, for a ten year contract, a ball park adjustment is of the order of about 50 basis points. In comparing the two discount factors from the two tables, the impact of the adjustment can be seen.

We now illustrate how ED futures can be used to hedge against unanticipated interest rate movements.

Hedging Against Interest Rate Increases

Consider, the following problem. A bank provides a firm a three month loan of $\$1m$ starting in 3 months at the LIBOR rate at the start of the loan. Let $t_0 = \frac{1}{4}$ be the start date and $t_1 = \frac{1}{2}$ the final payment date. The interest expense on this loan, due in 6 months is $\$1m \times \frac{\ell[t_0,t_1]}{4}$.

Lets consider what happens if the firm sells 1 ED futures contract, with an expiration date of 3 months. Ignoring the timing of cash flows due to marking to market, the profit from selling 1 ED futures contract at t_0 is $\pi(t_0)$, where

$$\pi(t_0) = \$1m \times [\frac{\ell[t_0, t_1]}{4} - \frac{I\ell_0[t_0, t_1]}{4}]. \tag{5.1}$$

The net effective interest expense is therefore:

$$\$1m \times \frac{\ell[t_0, t_1]}{4} - \$1m \times [\frac{\ell[t_0, t_1]}{4} - \frac{I\ell_0[t_0, t_1]}{4} = \$1m \times \frac{I\ell_0[t_0, t_1]}{4}.$$

Hence, by selling ED futures contracts, the firm exchanges an uncertain funding cost with a certain funding cost equal to the implied ED futures rate.

Actually, the above analysis is only approximate, since we ignored the timing of cash flows. First, the profit from the sale of the futures contract, given by equation (5.1), occurs at time t_0 while the interest expense on the loan occurs at time t_1. Second, we ignored the fact that with futures cash flows occur over the life of the contract, from date 0 to date t_0.

To fix the first timing problem, assume rather than selling 1 futures contract, we sold Q contracts. Then, if we assume that the profit (negative, if a loss) is invested in the ED market over the period $[t_0, t_1]$, at date t_1 it will have grown to:

$$\pi(t_1) = Q(1 + \frac{\ell[t_0, t_1]}{4}) \times \$1m \times [\frac{\ell[t_0, t_1]}{4} - \frac{I\ell_0[t_0, t_1]}{4}].$$

The net interest rate expense of the loan with hedging consists of the actual interest, less the profit from selling the futures, and is given by $C(t_1)$, where

$$C(t_1) = \$1m \times \frac{\ell[t_0, t_1]}{4} - \$1m \times Q(1 + \frac{\ell[t_0, t_1]}{4}) \times [\frac{\ell[t_0, t_1]}{4} - \frac{I\ell_0[t_0, t_1]}{4}]$$

Now, at date 0, if we knew the future LIBOR rate, $\ell(t_0, t_1)$, then by choosing $Q = \frac{1}{(1+\ell(t_0,t_1)/4)}$, the net interest expense with hedging would simplify to:

$$C(t_1) = \$1m \times \frac{I\ell_0[t_0, t_1]}{4}.$$

Unfortunately, at date 0, we do not know $\ell(t_0, t_1)$, so we cannot compute it exactly, but we may be able to estimate it well by the implied futures rate $I\ell_0(t_0, t_1)$. Then $Q = \frac{1}{(1+I\ell_0(t_0,t_1)/4)}$. Other than the issue of estimating Q, we have resolved the first part of the timing mismatch.

The second mismatch comes from the fact that we have really assumed the contract was a forward contract rather than a futures contract. Recall that futures contracts provide daily cash flows, over the period $[0, t_0]$, whereas forwards only provide this cash flow at the delivery date. The payoff from one forward contract with settlement date t_0 held over a day can be replicated by a trading strategy involving trading $P(0, t_0)$ otherwise identical futures contracts.

To see this, assume at date 0, a forward contract is sold. Assume further that $P(0, t_0)$ futures were purchased. The next day the $P(0, t_0)$ futures position is liquidated for a profit equal to:

$$P(0, t_0) \times (\text{ Change in Effective Futures Price}).$$

Now consider the change in value of a single forward contract held over one day. Since the change in the forward price is only obtained at the delivery date, the *value* associated with the change in forward price equals the present value of the change, or:

$$P(0, t_0) \times (\text{ Change in Effective Forward Price})$$

If rates were certain, forward prices changes would equal futures price changes, so the daily profit from holding $P(0, t_0)$ futures contracts equals the profit from holding 1 forward contract over one day. When interest rates are uncertain, assuming equality is only an approximation. Hence, to adjust for the marking to market feature rather than use 1 contract, we should use $P(0, t_0)$ contracts.

Combining the timing effect and the marking to market effects together, results in lowering the hedge ratio from 1 to a hedge ratio of $HR = Q \times P(0, t_0)$. Substituting for Q and taking $P(0, t_0) = \frac{1}{1+\ell(0,t_0)/4}$, we obtain:

$$HR = \frac{1}{(1 + I\ell_0(t_0, t_1)/4)} \times \frac{1}{1 + \ell(0, t_0)/4}$$

If we assume away the difference between implied futures rates and forward rates, then the right hand side represents the value at date 0 of a Eurodollar deposit that pays out \$1 at date t_1. That is

$$HR = P(0, t_1)$$

In summary, if ED futures were used to hedge against increasing LIBOR rates, rather than implementing a naive hedge, the number of ED futures to sell should be reduced by the present value factor between the current date, and the date where the interest rate loan is to be paid back. As time evolves the discount factor increases towards one, and the hedge strategy converges towards selling 1 futures contract.

Example: Using ED Futures to Hedge a Floating Rate Liability

(i) It is currently October 15th 2004. A firm plans on borrowing $100 million dollars on September 13th 2004 and pay back the loan after 3 months. The interest on the loan is tied to 3 month LIBOR at the start date. Interest rates are currently fairly low, and the firm is concerned that rates will rise dramatically. A naive hedge would be to sell 100 ED futures contracts that settle in September. Using the data from Table (5.3) this would lock in an implied futures rate of 1.53%. From Table (5.3) the appropriate discount factor to use is given by 0.988, so a slightly better hedge would involve selling 99 contracts. Note that as the time of the borrowing increases, and as interest rates increase, the difference between the naive and adjusted hedge increases.

(ii) A firm has floating rate liabilities of $1m indexed off 90 day LIBOR with payment dates that coincide with the maturity dates of ED futures. The firm is concerned that LIBOR rates will increase and would like to lock into a fixed set of rates using a naive hedge.

The current ED futures prices are given in Table 5.4. By selling a portfolio

Table 5.4 ED Futures Prices

Date	Futures Price	Implied LIBOR
March (t_0)	96.54	3.46
June (t_1)	96.35	3.65
Sept (t_2)	96.14	3.86
Dec (t_3)	96.00	4.00

of futures contracts with consecutive expiration dates, called a strip of futures, the firm can convert its floating rate liabilities into a sequence of fixed rate liabilities.

Assume the firm sells a strip of futures with March, June, September and December expiration dates. The actual LIBOR rates that occur on the expiration dates are shown in the second column of the table below.

Recall that the profit from selling one futures contract is $\$1m \times \left[\frac{\ell[t_0,t_1]}{4} - \frac{I\ell_0[t_0,t_1]}{4}\right]$.

Table 5.5 shows the differences between the actual and implied LIBOR rates and the profit on the sale of each contract. Since each basis point is worth $25, the final column is obtained by mutiplying the previous column by 25

Table 5.5 Profit form Sale of ED Futures Contracts

Date	Actual LIBOR	Basis Point Increase (LIBOR-Implied LIBOR)100	Profit on Futures
March (t_0)	4.00	54	1,350
June (t_1)	4.20	55	1,375
Sept (t_2)	4.20	34	850
Dec (t_3)	3.50	-50	-1,250

The final interest expenses of the unhedged and hedged positions are shown below.

Date	Unhedged	Hedged
t_0	Interest = 10,000	Interest = 10,000
		Hedging Cost = - 1,350
		Net Expense = 8650
	Effective Rate 4%	Effective Rate 3.46%
t_1	Interest = 10,500	Interest = 10,500
		Hedging Cost = - 1,375
		Net Expense = 9125
	Effective Rate 4.2%	Effective Rate 3.65%
t_2	Interest = 10,500	Interest = 10,500
		Hedging Cost = - 1,375
		Net Expense = 850
	Effective Rate 4.2%	Effective Rate 3.86%
t_3	Interest = 8,750	Interest = 8750
		Hedging Cost = 1,250
		Net Expense = 10000
	Effective Rate 3.5%	Effective Rate 4%

Notice that by hedging, the interest expenses are exactly equal to the implied ED futures LIBOR rates. The strip of futures converts the uncertainty of LIBOR rates into the set of certain ED implied futures rates.

To expedite the execution of strip trades, the CME offers bundles and packs. A bundle is a strip of futures contracts in consecutive series. A five year bundle, for example, consists of 20 ED futures contracts. Bundles are quoted on the basis of the net average price change of the contracts in the bundle. A pack is a bundle that consists of 4 contracts in each series, rather than 1.

Of course, in this example we have acted as if the futures contracts were forward contracts and we have not *tailed the hedge*. The consequences of tailing the hedge would result in a slightly smaller sale of strips.

5.5 ROLLING HEDGES

The above strategy of selling a strip of ED futures contracts to hedge a floating rate liability works well if there is sufficient liquidity in the distant ED futures contracts. Since ED futures contracts are liquid contracts, even for distant maturities, the trading of strips is very popular. If the hedge has to extend out over time periods where actively traded ED futures contracts do not exist, then an alternative hedging scheme can be established. In this case, a hedger can set up a rolling hedge. This is best illustrated with an example. As in our earlier example, we will, for the moment, ignore the difference between forward and futures contracts.

Example

A firm is scheduled to use 3 month borrowing for which it pays a rate linked to 3 month LIBOR. Assume the borrowing periods coincide with the settlement dates of succesive ED futures contracts. In particular, assume the firm's needs are 20m dollars in March, 40m dollars in June, 20m dollars in September and 50m dollars in December.

To hedge against LIBOR rates increasing, the firm could sell the appropriate strip of ED futures. A naive *one for one* hedge is

- sell 20 March ED futures
- sell 40 June ED futures
- sell 20 September ED futures
- sell 50 December ED futures.

A better dynamic hedge would require selling less ED futures. Assume the LIBOR discount function is given below:

Date of Loan	Time to Maturity	Discount Factor
March	1.0	0.962
June	1.25	0.951
Sept.	1.50	0.939
Dec.	1.75	0.919
March	2.00	0.900

In tailing the hedge, the number of ED futures contracts are reduced to $0.951 \times 20 = 19$ March contracts, $40 \times 0.939 \approx 37$ June contracts, $20 \times 0.919 \approx 18$ September contracts, and $50 \times 0.90 = 45$ December contracts. Over time these hedges need to be increased.

If the distant contracts were not that liquid, another strategy could be adopted. To get the basic idea, we first ignore the tailing, and reconsider a naive rolling hedge. The total number of dollars to be borrowed is $20m+40m+20m+50m = 130m$ dollars. At the initiation date, 130 March ED futures contracts are sold. Then, in early March, the position is liquidated, and $(130-20) = 110$ June futures contracts are sold. In early June, this position is rolled over into a short position in $110-40 = 70$ September futures. Finally, in September, this position is rolled over into $70-20 = 50$ Dec. futures.

A tailed hedge would initially consist of a short position in $0.951 \times 20 + 0.939 \times 40 + 0.919 \times 20 + 0.90 \times 50 \approx 120$ futures contracts. At the roll dates, the new hedge will be determined by the new discount factors. Actually, between roll dates, minor modifications to the hedge may need to be made.

ED deposits with different maturities enables the spot LIBOR curve to be constructed at the short end. The ED futures contracts provide us with information that will allow us to construct forward rates, and hence spot rates for short and middle maturities. Converting the futures rates to forward rates requires a convexity adjustment which we study in another chapter. There are other LIBOR based products that will allow us to construct a LIBOR zero curve for more distant maturities. These contracts are Forward Rate Agreements and Interest Rate Swaps.

5.6 FORWARD RATE AGREEMENTS

Forward Rate Agreements, or FRAs, are forward contracts on interest rates. Such contracts exist in most major currencies, although the market is dominated by US dollar contracts. The market is primarily an interbank market, and the major traders communicate their quotes via electronic quotation systems.

A FRA is a cash settled contract between two parties where the payout is linked to the future level of a designated interest rate, such as 3-month LIBOR. The two parties agree on an interest rate to be paid on a hypothetical "deposit" that is to be initiated at a specific future date. The buyer of an FRA commits to pay interest on this hypothetical loan at a predetermined fixed rate and in return receive interest at the actual rate prevailing at the settlement date.

Let $FRA_0[t_0, t_1]$ represent the annualized fixed rate determined at date 0, for the time period $[t_0, t_1]$. Let $\Delta t = t_1 - t_0$ be the time period in years.

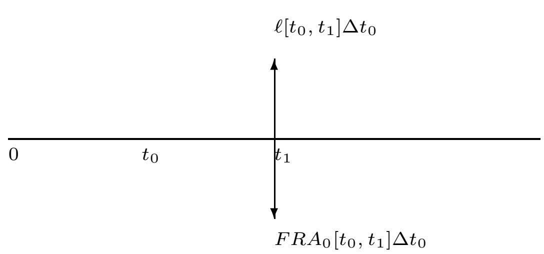
let $\ell[t_0, t_1]$ represent the reference rate, usually LIBOR, also in annualized form, that prevails at the settlement date, t_0. The net cash payment to the buyer of an FRA is based on a quantity, $Q(t_0)$, given by

$$Q(t_0) = (\ell[t_0, t_1] - FRA_0[t_0, t_1])N\Delta t$$

Here N is the hypothetical deposit quantity which is a predetermined fixed constant, usually referred to as the notional principal, and Δt represents the "deposit" period.

Actual settlement of this payment can occur in one of two ways. In the first form of FRA the actual cash payment of $Q(t_0)$ is made at date t_1. This contract is often preferred by corporations, but is not that common, and is shown in Figure 5.1

Fig. 5.1 Cash Flows from A FRA

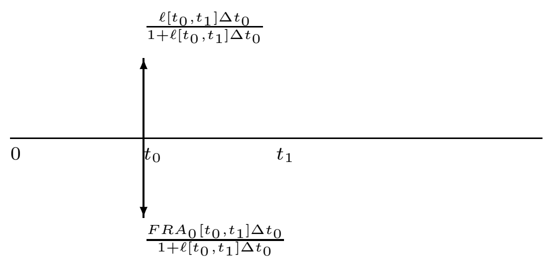


The more usual approach is to settle the contract at date t_0. In this case the actual cash flow is taken to be the present value of $Q(t_0)$, where the reference rate is used as the discount factor. That is, the cash payment at date t_0 is

$$\begin{aligned} c(t_0) &= Q(t_0)/[1 + \ell[t_0, t_1]\Delta t \\ &= \frac{\ell[t_0, t_1] - FRA_0[t_0, t_1]}{1 + \ell[t_0, t_1]\Delta t} \times N\Delta t. \end{aligned}$$

These cash settlememt contracts are more common among banks, and are shown in Figure 5.2

Fig. 5.2 Cash Flows from a FRA



Example

A bank buys a 3×6 FRA and read as “three by six”, with a notional of \$100m. This quote convention identifies the point in time when the contract begins (t_0 is 3 months) and ends (t_1 is 6 months). Assume the agreed FRA rate is 4%. ($FRA_0[t_0, t_1] = 0.04$) The buyer has committed to pay 4% on a hypothetical deposit that starts in 3 months and ends in 6 months. Assume the exact deposit period is 92 days and payment is based on “actual/360” day basis. In this case $\Delta t = 92/360$.

Assume three months later, three month LIBOR is at 6%. ($\ell[t_0, t_1] = 0.06$.) In this case, the bank will receive

$$c(t_0) = \frac{[0.06 - 0.04]\frac{92}{360}}{1 + 0.06(92/360)} \times 100m = \$503,392$$

FRA Prices and LIBOR Forward Rates

At the initiation date, the buyer and seller of a FRA agree on the fixed rate of the hypothetical deposit account. In particular, they establish $FRA_0[t_0, t_1]$ such that the value of the contract at date0 is 0. The actual value of $FRA_0[t_0, t_1]$ is given by the appropriate forward rate on the reference interest rate. In what follows we shall assume the reference rate is LIBOR.

To establish what the fair FRA price should be, consider the following strategy.

- Buy a discount bond that matures at date t_0. At maturity roll the \$1 face value into a Eurodollar deposit that matures at date t_1.

This guarantees $1 + \ell[t_0, t_1]\Delta t$ dollars at date t_1.

- Sell a discount bond that matures at date t_1. This guarantees a one dollar obligation at date t_1, that can come from funds from the above investment.

  This leaves $\ell[t_0, t_1]\Delta t$ dollars at date t_1.

- Sell a FRA with cash settlement date t_0.

At t_0 the payout of the FRA (with \$1 Notional) is

$$\frac{FRA_0[t_0, t_1] - \ell[t_0, t_1]}{1 + \ell[t_0, t_1]\Delta t}\Delta t.$$

Assume these dollars are placed in a ED deposit with maturity t_1. The value grows to

$$(FRA_0[t_0, t_1] - \ell[t_0, t_1])\Delta t$$

Adding these proceeds to the previous funds lead to a guaranteed cash flow of $FRA_0[t_0, t_1]\Delta t$ dollars at date t_1. The table below summarizes the cash flows:

Strategy	Cash Flow at Date 0	Cash Flow at Date t_1
Buy 1 bond that matures at date t_0 and roll the proceeds into a ED account at date t_0.	$P(0, t_0)$	$1 + \ell[t_0, t_1]\Delta t$
Sell 1 bond that matures at date t_1	-$P(0, t_1)$	-1
Sell a FRA	0	$-(\ell[t_0, t_1] - FRA_0[t_0, t_1])\Delta t$
Net Cash Flows	$P(0, t_0) - P(0, t_1)$	$FRA_0[t_0, t_1]\Delta t$

The cost of this strategy at date 0 is $P(0, t_0) - P(0, t_1)$ dollars. If this amount was financed for the period $[0, t_1]$, the amount owed at date t_1 would be

$$\frac{P(0, t_0) - P(0, t_1)}{P(0, t_1)} = \frac{1}{FO_0[t_0, t_1]} - 1 = f_0[t_0, t_1]\Delta t.$$

The net profit at date t_1 would be

$$(FRA_0[t_0, t_1] - f_0[t_0, t_1])\Delta t$$

and the net initial investment would be \$0. Clearly, to avoid riskless arbitrage, the final value should also equal \$0. The fair FRA price is

$$FRA_0[t_0, t_1] = f_0[t_0, t_1]$$

That is, the fair FRA rate is the forward rate for the period.

Example

The current three month LIBOR is $\ell[0, t_0] = 0.06$, where $t_0 = 92$ days. The current six month LIBOR is $\ell[0, t_1] = 0.06$ where $t_1 = 182$ days. We want to compute the fair price of a 3×6 based on an "actual/360 day" basis.

First, $\Delta t = \frac{182-92}{360} = 0.250$. Now,

$$FRA_0[t_0, t_1]\Delta t = \frac{P(0, t_0)}{P(0, t_1)} - 1$$

Substituting

$$\begin{aligned} P(0, t_0) &= \frac{1}{1 + 0.06(92/360)} = 0.98489 \\ P(0, t_1) &= \frac{1}{1 + 0.06(182/360)} = 0.97055 \end{aligned}$$

into the above equation leads to a FRA rate of 5.910%. Notice that if the date basis was "Actual/365", then $\Delta t = \frac{90}{365} = 0.24657$ years.[2]

FRA quotes on LIBOR are therefore precisely the same objects as forward rates. Usually, the LIBOR zero curve, and hence the FRA quotes, is established using Eurodollar futures prices and interest rate swap rate data. Eurodollar futures are often used to hedge and price FRA contracts.

5.7 EURODOLLAR FUTURES VERSUS FRAS

Notice that a firm that *buys* a FRA gains from interest rate *increases*. In contrast, a long position in a ED futures contract gains if interest rates *decline*. Buying a FRA is almost identical to selling a ED futures contract. Of course, since FRAs are forward contracts they are not marked to market daily as ED futures. Banks can use ED futures to hedge exposed FRAs. Unlike exchange traded futures, however, the FRA can be customized to closely conform to the specific risk being hedged by the firm.

[2]FRAs in British pounds use this convention. Most other currencies use "Actual/360" day convention.

5.8 PRICING FRAS USING ED FUTURES

In practice, due to the daily marking to market feature of futures contracts, FRA rates will be lower than those implied by ED Futures rates. For the moment, we will ignore the differences between futures and forwards. In this section, we will learn how to calculate FRA strip rates from a yield curve consisting of a spot ED deposit rate up to the first ED futures setllement date (the cash stub rate) and the set of ED futures prices. Adjustments for the convexity affect, and for other nuances, like the *turn of the year* effect will be considered later.

To make matters specific consider the data from Table (5.3) reproduced in the table below.

Contract	Value Date	Days	Rate
Stub	1/15/04	60	1.16
March 04	$t_0 = 3/15/04$	91	1.16
June 04	$t_1 = 6/14/04$	91	1.28
Sept 04	$t_2 = 9/13/04$	91	1.53
Dec 04	$t_3 = 12/13/04$	91	1.89

From the table we also obtain:

$$\begin{aligned}
P(0,t_0) &= \frac{1}{1+0.0116(60/360)} = 0.99807 \\
P(0,t_1) &= P(0,t_0) \times \frac{1}{1+0.0116(91/360)} = 0.99515 \\
P(0,t_2) &= P(0,t_1) \times \frac{1}{1+0.0128(91/360)} = 0.99194 \\
P(0,t_3) &= P(0,t_2) \times \frac{1}{1+0.0153(91/360)} = 0.98812
\end{aligned}$$

Actually, to be more precise we should have convexity adjusted the futures rates and used the resulting forward rates (and discount factors) in Table (5.2). These values are:

$$\begin{aligned}
P(0,t_0) &= 0.99807 \\
P(0,t_1) &= 0.99515 \\
P(0,t_2) &= 0.99195 \\
P(0,t_3) &= 0.98345.
\end{aligned}$$

Once the discount factors are obtained then the FRA values can be identified, using the equation

$$FRA_0[t_i,t_{i+1}]\Delta t_i = \frac{P(0,t_i)}{P(0,t_{i+1})} - 1 \text{ for i = 0,1,...}$$

where $\Delta t_i = \frac{t_{i+1}-t_i}{360}$ if t_i and t_{i+1} are in days, and the basis is "actual/360".

To determine the discount bond price for a given date that lies between two futures dates, requires interpolating values. Assume, $t_1 < t < t_2$. One way of approximating $P(0,t)$ is:

$$P(0,t) = P(0,t_1)\frac{1}{(1+FRA_0[t_1,t_2]\Delta t_1)^{\frac{t-t_1}{t_2-t_1}}}$$

To illustrate the idea, consider a 3×6 FRA starting from $s_0 = 3/20/04$ and expiring on $s_1 = 6/20/04$. The number of days between s_0 and s_1 is 92 days In order to establish this FRA price we need the discount factors for these two dates.

$$\begin{aligned} P(0,s_0) &= 0.99807 \times \frac{1}{(1+0.01597(91/360))^{\frac{5}{91}}} = 0.99784 \\ P(0,s_1) &= P(0,t_1) \times \frac{1}{(1+0.012876(91/360))^{\frac{6}{91}}} = 0.994937 \end{aligned}$$

Then, substituting into the following equation,

$$FRA_0[s_0,s_1]\frac{92}{360} = \frac{P(0,s_0)}{P(0,s_1)} - 1,$$

leads to $FRA_0[s_1,s_2] = 0.011454$ or 1.1454%.

Adjusting FRA Prices

The above analysis has incorporated the convexity correction that is needed to adjust for the fact that the data came in the form of futures rates not forward rates. A second adjustment is necessary if the FRA extends over a calendar year. It has been observed historically, that at the year end there is a scarcity of funds and short term interest rates often rise for the the last business day in the calendar year to the first business day in the new year. As trading commences in the new year, rates return to prior levels. It therefore is common practice to make adjustments in the December interest rate futures contract for this year end turn affect.[3]

5.9 CONCLUSION

In this chapter we have examined ED deposits, ED futures and the over the counter market for FRAs. Given ED spot rates over the short end of the

[3]For details on the Year end turn effect see Burhardt and Hoskins article in Risk Magazine 1997???

curve, the LIBOR spot rate to the expiration date of the nearest ED futures contract, and given the prices of successive ED futures contracts, it is possible to set up a LIBOR zero curve extending out beyoond five to seven years. In the next chapter we shall see that interest rate swaps contain information on PAR LIBOR rates that can be used to extract spot LIBOR rates for longer term contracts. In a later chapter we will examine how one adjusts the implied LIBOR ED futures rates into forward rates using a convexity adjustment. Given the LIBOR spot curve, FRA prices can easily be established as the appropriate forward rate.

This chapter has also illustrated the potential uses of FRAs and/or ED futures. In particular, these contracts allows a firm to replace floating interest rates with fixed interest rates or vice-versa. FRAs are customized contracts that can be obtained through investment banks. These banks hedge the risk of these products by using ED futures. In hedging the sale of a forward contract with futures, the marking to market feature of futures has to be taken into account. This involves tailing the hedge. Overall, very effective hedges can be put into place. As a result, the pricing of FRAs is very competitive and bid-ask spreads are very narrow.

Exercises

1. Mechanics of ED Futures Contracts.

   A firm borrows 100 million dollars for one year, begining on September 17th. Spot three month LIBOR on that date is 5%. The firm must pay 150 basis points above LIBOR. So the initial interest rate is 6.5%. This rate is for the first 3 months. Subsequently, the LIBOR rate will be reset to the then current rate, with the spread staying fixed at 150 basis points. The reset dates are December 17th of this year, March 17th and June 17th. On September 17th, the following ED futures prices are observed.

Delivery Month	Futures Price
December	95.34
March	95.56
June	96.00
September	95.5

   (a) Compute the implied ED futures LIBOR rates for each of the quarters in the year.

   (b) Ignoring the difference between futures and forward rates, use the results of (a) to compute the LIBOR discount function from September 17th to December, March, June, September and to the following December. Assume that there are exactly 90 days in each quarter and 360 days in the year.

   (c) What ED Futures trades should be used to construct a naive hedge that will hedge against rising LIBOR rates?

   (d) Adjust the naive hedge so as to take into account the timing of cash flows.

   (e)Suppose the following spot LIBOR and Futures prices were observed.

Date	Spot LIBOR	December Futures	March Futures	June September
December	4.00%	96.00	95.00	94.82
March	4.50%	-	95.50	96.60
June	4.80%	-	-	95.20

   Compute the interest expenses each quarter for an unhedged position. Compare these interest rate expenses to those derived from a strip. Finally set up a naive rolling hedge using the short dated futures contracts and compute the interest expenses assuming all trading took place at the above dates.

(f) How could the rolling hedge be adjusted? In particular compute the initial number of short dated ED futures contracts that should be traded.

2. Mechanics of FRAs

Consider a \$1m 3×6 FRA quoted at 5.0%. A firm buys this contract. After 3 months the spot LIBOR rate is 5.2%. Compute the profit on the FRA assuming that there are 91 days in the final three month period.

3. Hedging FRAs with ED Futures.

Consider the problem of hedging a 100m dollar 2×5 FRA at 8.5%.

(a) Write down the cash flow of the FRA that occurs in 2 months time, as a function of LIBOR rates at that time.

(b) Assume the current 2 month (60 day) LIBOR spot rate is 8.1%. Assume further that in this FRA there are 92 days between months 2 and 5. Establish the present value of a basis point change in the FRA agreement.

(c) The value of a basis point change in the ED futures contract is \$25. Using (b) establish the number of futures contracts that are required to hedge the purchase of the above FRA.

(d) All things being equal, what will happen to the hedge ratio over time.

4. Computation of a Discount Function using ED Futures

The following information is given. The spot LIBOR rate for the first 23 days is 5.0%. The table below shows the expiration dates of successive ED futures contracts and the corresponding implied ED futures 3 month LIBOR rate.

Maturity	Implied 90 day ED futures LIBOR rates
23	5.10
114	5.2
205	5.30
296	6.0
397	6.25
489	6.50

(a) Ignoring the difference between futures and forwards compute the LIBOR discount function, unadjusted by a convexity correction.

(b) Use the discount function to establish a fair Forward rate agreement rate for a contract that matures in 296 days, and is based on 3 month LIBOR.

5. Hedging Floating Rate Liabilities

   A bank funds itself with 3 month ED time deposits at LIBOR. The current LIBOR rate is 5.5%. The successive 3 month LIBOR rates are unknown of course. However, the implied ED futures rates of contracts maturing in months 3, 6 and 9 are 5.8%, 6.05%, and 6.20%. Assume each of these three month intervals contain exactly 91 days.

   The bank has a customer who wants a 100m dollar loan, with fixed interest paid quarterly, at dates that correspond to the expiry dates of the ED futures.

   (a) Explain the risk the bank takes in providing a firm a fixed rate loan.

   (b) Explain how the bank can use ED futures to swap its floating rate exposure into a fixed rate exposure.

   (c) Ignoring the difference between forward and futures, establish the fair fixed rate the bank could establish for its client.

   (d) What hedging strategy should the firm set up at date 0. Make sure you tail your hedge. Explain any limitations of this analysis, and this hedge. In particular, explain the direction of bias, introduced by not taking into account the difference between forwards and futures.

6. Set up a FRA calculator in Excel, that has the following structure:

Inputs:

Settlement Date
Underlying Maturity Date

Spot Rate to FRA Expiration Date (%)
Interest Rate Frequency (1 (annual) , 2 (semi-annual), or 4 (quarterly))

Spot Rate to Final Maturity Date (%)
Interest Rate Frequency (1,2, or 4)

Intermediate Calculations:

Days to Expiration Date
Days to Underlying Maturity Date
Days Between Maturity Date and Expiration Date
Continuous Rate to Expiration (%)
Continuous Rate to Maturity (%)

Output:

Forward Rates (%):
-Continuous Compounding
-Quarterly Compounding
-Semiannual Compounding
-Annual Compounding

In the spreadsheet, the interest rate frequency refers to the compounding interval and is 4 if quarterly, 2 if semiannual, and 1 if annual.

7. FRAs and the Cost of Carry Model.

   LIBOR rates are shown below:

Maturity	Rate
7 days	2.00
14 days	2.20
30 days	2.30
60 days	2.50
120 days	3.00

   (a) Use these LIBOR rates to construct the price of LIBOR bonds with maturities 7, 14, 30 and 60 days.

   (b) Establish the theoretical ED futures price for a 90 day ED futures contract that expires in 30 days time. Ignore the difference between futures and forwards.

(c) If the actual price of the ED futures was higher than the theoetical price by 30 basis points, how could you set up an arbitrage free position. What assumptions have you made?

(d) Establish the fair FRA value for a contract that starts in 7 days and ends in 90 days. What assumptions have you made.