Time Value of Money (TVM)

A dollar today is more valuable than a dollar sometime in the future......

- The intuitive basis for present value – what determines the effect of timing on the value of cash flows?
- Different types of cash flows, and ways to estimate present value of each of these types.
- Applications of present value in corporate finance.

If you are given the choice of taking $1000 today versus $1000 in one year, which one will you pick, and why?

The power of compounding – an example:

In 1626, Peter Minuit bought the entire Manhattan Island from the native Americans for about $24 in goods. Suppose they had sold the goods and invested $ 24 at 8%. What would it be worth today?

- $ 71 million
- $ 71 billion
- $ 71 trillion
- More!

Present Value (PV) is the simplest and most powerful tool in finance.

Time line:

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|_____ |_____ |_____ |.........................|_____ |
0        1        2        3                        n-1     n

PV                                                   FV
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- Time lines are very useful in solving TVM problems.
- Distinction between *point in time* and *period of time.*
- Cash flow at time *0* is already in PV terms, so does not need to be discounted.
- Only cash flows at the same *point in time* can be meaningfully compared or aggregated.

Intuitive Basis for Present Value

- Individuals prefer present consumption to future consumption. Hence, they would have to be offered more in the future to give up present consumption.
- The value of currency decreases due to inflation – hence a dollar tomorrow would be worth less than a dollar today.
- Any uncertainty (risk) associated with cash flows in the future reduces the value of the cash flow.

All of these three reasons are different, and independently make current cash flows more valuable than future cash flows.

Discount Rate:

The rate at which present and future cash flows are traded off. It incorporates all of the three reasons for present valuing cash flows:

- The preference for current consumption (*greater preference ⇒ higher discount rate*)
- Expected inflation (*higher inflation ⇒ higher discount rate*)
- Uncertainty in future cash flows (*higher risk ⇒ higher discount rate*)

Also called interest rate, rate of return, hurdle rate, cost of capital, opportunity cost of capital, etc.

Basic Types of Cash Flows

- Simple cash flows – these are cash flows in specified future time periods.

- Annuity – a constant cash flow that occurs at regular intervals for a fixed period of time.

- Growing annuity – a cash flow growing at a constant rate (*the growth rate can be negative also!*), for a fixed period of time.

- Perpetuity – a constant cash flow at regular intervals *forever*.

- Growing perpetuity – a cash flow growing at a constant rate forever.

Discounting and Compounding

Discounting – computing the present value of a cash flow:

$$PV = \frac{CF_t}{(1+r)^t}$$

E.g., suppose you do a consulting project in return for receiving $50,000 after one year. The present value of this contract to you is (assume a discount rate of 8%):

50,000 / (1+0.08) = 46296.29

Discounting is moving future cash flows to a present date

$\frac{1}{(1+r)^t}$ is called the *present value interest factor* $\text{PVIF}_{r,t}$

Compounding – computing the future value of a cash flow:

$$FV = CF_0(1+r)^t$$

E.g., suppose you invest $1,000 today at an interest rate of 5%, for one year. How much will you receive at the end of one year?

1,000(1+0.05) = 1050

Compounding is moving cash flows to a future date

$(1+r)^t$ is called the *future value interest factor* $\text{FVIF}_{r,t}$

Calculation Methods

Three methods of calculation:

- Formulae
- Tables
- Financial Calculator

Example: To compute PV of $50,000 to be received one year from today.

Using Formula:

PV = 50,000 / (1+0.08) = 46296.29

Using Tables:

PV = 50,000 x PVIF(1,8%)

= 50,000 x 0.9259 = 46,295

Using Financial Calculator:

FV = 50,000, n=1, I=8, press PV key

PV = 46,296.29

Same way for future value calculations.

An interesting short cut

How many years does it take for money to double, if interest rate is:

6% p.a.	yrs
7%	yrs
8%	yrs
9%	yrs
10%	yrs
11%	yrs

Do you see a pattern?

Its called the rule of 72 – remember, its only an approximation, and it works well only for interest rates not too far away from 8%.

Frequency of Compounding/Discounting

If interest rates are computed more frequently than annually (monthly, daily, etc.), the PV or FV can be quite different from those computed on an annual compounding basis – the *effective* annual interest rate (EAR) can be very different from the *stated* annual interest rate (also called APR – annual percentage rate).

$EAR = \left(1 + \frac{APR}{n}\right)^n - 1$, where n =number of compounding periods during the year

Extreme Case: Continuous Compounding
Limiting case is to compound every infinitesimal instant (banks and financial institutions use this rate very often!). In this case,

$$EAR = e^{APR} - 1, \qquad e=2.718 \text{ (approx.)}$$

General Rule: The higher the APR, more is the effect of increasing compounding frequency.

Two banks can charge the same nominal APRs, but their EARs may be quite different depending upon compounding frequency.

Example: MonthComp bank charges 8%, compounded monthly, while SemiComp bank charges $8\frac{1}{8}$% compounded semi-annually. Which bank would you go to for a new loan?

Annuities

An *annuity* is a series of constant cash flows that occur at the end of each time period for some fixed number of periods.

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|____ |____ |____ |......................|____ |
0     1     2     3                  n-1   n
      C     C     C                  C     C
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Present value of an *end-of-period* annuity:

$$PV(Annuity) = C\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right]$$

Or, using tables,

PV (Annuity) = C x PVIFA(n,r)

Example: Many sports compensation contracts are structured as annuities – how much is Derrick Coleman's (NJ Nets) $69 mil 10-yr contract worth in PV terms, assuming a 10% discount rate?

Can reverse the equation to compute the annuity given a present value.

Examples: House mortgage payments, Car loan payments. What would be the monthly mortgage payment on a 7¾%, 15 yr. fixed mortgage for a $250,000 house with 10% down?

Future value of an *end-of-period* annuity:

$$FV(Annuity) = C\left[\frac{(1+r)^n - 1}{r}\right]$$

Or, using tables,

FV (Annuity) = C x FVIFA(n,r)

Example: Retirement Accounts (IRAs). What would be the value of your account at age 65, if, from age 30 onwards, you contribute $10,000 every year, and the annual return is 8%?

Again, can reverse the equation to compute the annuity given a future value.

Example: Sinking fund provision for a bond. How much would a company need to set aside each year for bonds with a face value of $100 mil, coming due in 10 years, assuming an 8% p.a. return?

For *beginning-of-the-period* annuities, the formulae are modified:

$$PV(Annuity) = C + C\left[\frac{1 - \frac{1}{(1+r)^{n-1}}}{r}\right]$$

$$FV(Annuity) = C(1+r)\left[\frac{(1+r)^n - 1}{r}\right]$$

Growing Annuities

A cash flow that grows at a *constant* rate for a specified period of time.

$$PV(Growing\ Annuity) = C(1+g)\left[\frac{1-\frac{(1+g)^n}{(1+r)^n}}{r-g}\right]$$

where *g* is the constant growth rate in cash flows. When *g*=*r*, PV=nC.

Example: Suppose you have rights to a gold mine for the next 20 years, over which you plan to extract 5000 ounces of gold every year. The current price per ounce is $300, but is expected to increase at 3% per year. If the discount rate is 8%, how much should you be willing to pay for these rights?

Question: If both the growth rate and the discount rate increase by 1%, how is the PV affected?

Perpetuities

A constant cash flow at regular intervals forever.

Does it ever happen? Yes:

- Console bonds in the UK
- Stock valuation!

$$PV(Perpetuity) = \frac{C}{r}$$

$$PV(Growing\ Perpetuity) = \frac{CF_1}{(r-g)}$$

In a growing perpetuity, the growth rate has to be less than the discount rate, for a finite PV.

Example: Stock Valuation – Eastman Kodak paid a dividend of 44 cents in the most recent quarter. Their earnings and dividends are expected to grow at 4% per year in the long run. If their average cost of capital is 12%, what is the value of the stock?

(*How does it compare with the closing price of the stock of $45.89 on August 28, 2001?*)

The Real World

- The cash flows are rarely as neat as any of the examples above.
- Usually cash flows are uneven – the cash flows, or their growth rates, can change from period to period (in fact, discount rates may also be different over different periods).
- The way to proceed is:
    - draw a time line of cash flows.
    - recognize the familiar components of the cash flows, like annuities, growing annuities, etc.
    - compute the PV of each component separately (or FV, as the case may be).
    - then aggregate cash flows, *after* they have been brought to the same date.

Example: Continuing the Eastman Kodak case, suppose their earnings and dividends are expected to grow at 6% for the first 5 years, and then at 4% forever, what should be the value of the stock (12% cost of capital, $1.76 last quarter dividend)?

(*This is a sum of a growing annuity and a growing perpetuity – be careful about discounting all cash flows back to date 0*)

A Note on Discount Rates

Different individuals, even when they agree on the cash flows, may come up with different discount rates, because:

- The preference for current consumption over future consumption, which determines the real rate of return, may vary across individuals.
- Inflation expectations, which determine the nominal rates, may vary.
- The degree of risk aversion may vary across individuals, resulting in different premia attached to the uncertainty associated with cash flows.

Does that mean that we all should use different discount rates?

In the presence of competitive financial markets in which investors can lend and borrow, the risk of a project or an investment can be evaluated independently of the risk-aversion characteristics of the investors in that project or investment – this is called the *Separation theorem.*

Hence, the discount rate can be separated from individual preferences, and is determined by arbitrage considerations.

The presence of a market interest rate (the rate at which the demand for funds equals the supply), enables individuals to adjust their consumption patters to meet their preferences, while accepting the same rate.