Bond Valuation

What is a bond?

When a corporation wishes to borrow money from the public on a long-term basis, it usually does so by issuing or selling debt securities called *bonds*. A bond is normally an interest-only loan, meaning that the borrower will pay the interest every period, but none of the principal will be repaid until the end of the loan.

Bond terminology

- *Par value (face value)*: Stated face value of the bond. It is usually $1,000.
- *Maturity date*: The date on which the par value must be repaid.
- *Interest payment (coupon payment)*: The fixed amount of interest usually paid every 6 months.
- *Coupon rate*: The interest payment divided by the par value. e.g. $\frac{80}{1,000}$=8%.
- *Call provision*: The provision whereby the issuer may pay bonds off prior to maturity.

The bond market is huge. At the end of 1994, the face value of bonds outstanding in the 22 largest international markets totaled $18.5 trillion, according to Salomon Brothers. That was 38% larger than the $13.4 trillion value of stocks outstanding worldwide.

Bond valuation model

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots\ldots + \frac{C}{(1+r)^n} + \frac{F}{(1+r)^n}$$
$$= C\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right] + \frac{F}{(1+r)^n} = C \times PVIFA(r,n) + F \times PVIF(r,n)$$

PVIFA: present value interest factor for annuity (A.2).
PVIF: present value interest factor for a lump sum (A.1).

Example: A 10% coupon bond has ten years to maturity and $1,000 face value. If the required rate of return for this bond is 10%, how much does this bond sell for?

- Method 1: Use tables A.1 and A.2.
  PV=100×6.1446+1,000×0.3855=$1,000
  A par bond (i.e., the bond is sold at its par).

- Method 2: Financial calculator
  Inputs: n=10, i =10, PMT=100, FV=1,000
  press the PV key, PV=-1,000.

- If market interest rate (r) increases to 12% right after the bond has been issued, how does this affect the bond price?

  $PV=100\times 5.6502+1{,}000\times 0.322=\$887<\$1{,}000$

  A discount bond (the bond is sold below par).

Bondholders earn less than that offered by the market, hence the bond must sell at a lower price.

Discount=\$1,000-887=\$113.

- If r decreases to 8% right after the bond has been issued, how does this affect the bond price?

  $PV=100\times 6.7101+1{,}000\times 0.4632=\$1{,}134.20>\$1{,}000$

  A premium bond (the bond is sold above par).

Bondholders earn more than that offered by the market, hence the bond must sell at a higher price.

Premium=\$1,134.20-1,000=\$134.20.

- Bond interest rate risk: bond prices fluctuate as the interest rate changes – *increase* when interest rates *decrease*, and vice versa (inverse, *convex* price-yield relationship).

Bonds with semiannual coupons

- Convert C, n, and r into $C/2$, $2n$, and $r/2$.

$$PV = C/2[PVIFA(r/2, 2n)] + F[PVIF(r/2, 2n)]$$

Example: A bond has an annual coupon rate of 15%, but coupons are paid semiannually. If the required rate of return on the bond is 10%, and the bond has 15 years to maturity, what is the bond price today?

$$\begin{aligned} PV &= 75[PVIFA(5\%, 30)] + 1{,}000[PVIF(5\%, 30)] \\ &= 75 \times 15.3725 + 1{,}000 \times 0.2314 \\ &= \$1{,}384.34. \end{aligned}$$

Yield to maturity (YTM)

Also called required rate of return / market rate / internal rate of return.

YTM: The interest rate required in the market on a bond.

- Finding the YTM: Trial and Error

Example: An 8% coupon bond with 6 years to maturity is selling for $912.92. What is the YTM of the bond?

At r=8%, PV=$1,000, par bond.
Since PV=$912.92, YTM>8%. Why?

- Try r=9%. PV=80×4.4859+1,000×0.5963=955.17.
- Try r=10%. PV=80×4.3553+1,000×0.5645=912.92.
- Hence, YTM=10%.

Financial calculator inputs:
n=6, PV=-912.92, PMT=80, FV=1,000
press the i key, i=10%.

Other Yield Measures

- *YTM*: The compounded interest rate that makes the present value of the cash flows equal to its price.
  - *assumes all coupon interest payments are reinvested at the Yield to Maturity (reinvestment rate risk).*
- *Current Yield*: Annual dollar coupon interest per unit price of the bond.

  E.g. 15-yr 7% coupon bond, $1000 par, selling for $769.40

$$current - yield = \frac{\$70}{\$769.40} = 9.10\%$$

  - *ignores capital gain/loss*
  - *ignores time value of money*
- *Yield to Call*: For bonds that may be called prior to the stated maturity date, *YTC* is the yield of the bond assuming it's called on its first call date, at the call price.

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots\ldots + \frac{C}{(1+r)^{n*}} + \frac{M^*}{(1+r)^{n*}}$$

  E.g. 18-yr 11% (semi-annual) coupon bond, $1000 par, selling for $1168.97. Suppose first call date is 13 yrs from now, and the call price is $1055.

  here, YTC = 9.2%

  - *YTM assuming bond is called on first call date.*

How are bond prices reported?

- *Most* Corporate bonds are traded in the OTC (Over-The-Counter) markets.
- Government Bonds (Treasury securities) are auctioned, and then traded in secondary markets by *dealers*.

Inflation-indexed Bonds

On Jan 29, 1997, the Treasury Department issued $7 billion of the so-called inflation-indexed bonds.

Key features:

- The principal amounts are adjusted for inflation annually.
- Regular semi-annual interest payments are also adjusted for inflation although the interest rate is fixed for the entire life.
- 10-year notes with face value of $1,000

Example: Suppose that you buy a $1,000 note at the beginning of the year and the interest rate on the notes was set at auction at 3%. Inflation that year is 3%.

- At the end of the first year, the $1,000 principal becomes $1,030, adjusted for inflation—although you won't receive any of that increased amount until the maturity. At the end of the first year, the notes will pay 3% interest on the $1,030 principal.

- If you hold your note until maturity, and inflation remains 3%, the principal will be

  $$FV = 1{,}000(FVIF_{3\%,10}) = \$1{,}344.$$

  However, this inflation-adjusted amount represents interest, and thus is taxable.

Common Stock Valuation

Common stocks are the units of ownership of a public corporation. Owners are entitled to receive dividends and capital appreciation as well as voting powers.

What is the difference between a bond and a stock?

- Ownership
- Voting rights
- Dividends
- Residual claim
- Limited liability

The general model

Cash payoff from stocks (in terms of future cash flows):

- cash dividends,
- capital gains / losses (upon selling the stock).

Expected return by holding a stock for one period is:

$$r = \frac{DIV_1 + P_1 - P_0}{P_0}.$$

hence, $$P_0 = \frac{DIV_1 + P_1}{1+r}$$

In the same way, the price at the end of the first period can be expressed as:

$$P_1 = \frac{DIV_2 + P_2}{1+r},$$

By iteratively substituting for P_1, P_2, etc., we get:

$$\begin{aligned} P_0 &= \frac{DIV_1 + \frac{DIV_2 + P_2}{1+r}}{1+r} \\ &= \frac{DIV_1}{1+r} + \frac{DIV_2 + P_2}{(1+r)^2} \\ &= \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{DIV_3}{(1+r)^3} + \ldots\ldots\ldots\ldots \\ &= \sum_{t=1}^{\infty} \frac{DIV_t}{(1+r)^t}. \end{aligned}$$

Thus, the value of a stock is equal to the present value of all expected future dividends.

Constant growth stocks: The Gordon Model

- If the dividend growth rate, *g*, is assumed to be constant, then:

$$P_0 = \frac{DIV_1}{r-g}.$$

Example: DIV_0=\$2.00, r=10%, g=5%,

$$P_0 = \frac{2(1+5\%)}{10\%-5\%} = \$42.00.$$

- $r = \dfrac{DIV_1}{P_0} + g$, i.e.

  total return=dividend yield+capital gains yield

  In the above example:

  dividend yield = 5%

  capital gain yield = 5%

  Double check:

$$P_1 = \frac{DIV_2}{r-g} = \frac{2(1+5\%)^2}{10\%-5\%} = \$44.1.$$

$$\text{Capital gains yield} = \frac{44.1-42}{42} = 5\%,$$

$$\text{Dividend yield} = \frac{DIV_2}{P_1} = \frac{2(1+5\%)^2}{44.1} = 5\%.$$

Hence, for a constant growth stock:

- Stock price grows at the dividend growth rate *g*, which is the same as the capital gains yield.
- Dividend yield remains constant at *r-g*.

How do you estimate *g*?

It's a billion-dollar question in valuation!

Can estimate by the following ways:
- Historical (past) growth as a measure of expected growth,
- Trust analysts and their projections,
- Put your faith in fundamentals.

What determines a firm's growth rate?
- Firm's reinvestment policy,
- Firm's project quality.

Expected growth in earnings

$$= \text{Reinvestment Rate x Return on Investment}$$
$$= \text{Retention Ratio x ROE}$$
$$= \text{(1-payout ratio) x ROE}$$
$$= (1 - \frac{DPS}{EPS}) \text{ x } \frac{EPS}{BPS}$$

(*Why should this hold? Why ROE?*)

Now, the growth rate in dividends equals the growth rate in earnings *if* the payout ratio is constant (*is it?*).

So what we need is assumptions about (and estimates of):
- Reinvestment rates,
- Marginal returns on equity.

How do you estimate how long the expected growth will last?

- The greater the current growth rate in earnings of the firm, relative to the stable growth rate (that can be sustained *forever*), the longer the high growth period.
- The larger the current size of the firm, the shorter the high growth period – size pushes firms towards stable growth.
- The greater the barriers to entry in a business, the longer the length of the high growth period for the firm.

At the end, you have to make a judgement call – as much an art as it is a science!

What should be the stable growth rate?

- No firm, in the long term, can grow faster than the economy it is in (*why not?*).
- So, stable growth rate cannot be greater than the growth rate of the economy.
- Stable growth rate cannot be greater than the discount rate, because the riskfree rate that is embedded in the discount rate will also build on these same factors – real growth in the economy and the expected inflation rate.
- Empirical fact: Very few firms (*Microsoft?*) are able to sustain high growth periods longer than 10 years.

Supernormal growth stocks

Example: A firm has been growing at a rate of 11% per year in recent years. This same growth rate is expected to last for another 3 years, then the growth rate will be back to the normal rate of 4% per year forever. If DIV_0=$5.00, r=13%, what is the firm's stock worth today?

The rule: If dividends grow steadily after t periods, the stock price can be written as:

$$P_0 = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \dots\dots\dots + \frac{DIV_t}{(1+r)^t} + \frac{P_t}{(1+r)^t} \quad .$$

Why discount Dividends, why not earnings?

- Investors receive dividends, not earnings!
- Some part of earnings has to be retained and invested, to generate future returns. Discounting earnings would ignore the investment needed to generate these returns.

How do you value no-dividend firms? As per the Gordon Model, they should be valued at zero!

- A firm with many attractive growth opportunities may not want to declare dividends, and invest all earnings into projects.
- This would increase the value of the firm.
- Hence, investors would buy the stock at positive prices with the hope of:
  - selling it for higher later, as the increase in firm valuation would increase stock price.
  - receiving cash or shares of stock of the acquirer, if the firm is acquired (if it has attractive growth opportunities, it could be an attractive acquisition candidate!).
  - receiving bonus shares if the stock splits.

These hypotheses are borne out by empirical evidence:

- High growth firms pay lower dividends (Microsoft?).
- Utility firms have high payout ratios, as they do not have many growth opportunities.

Present value of growth opportunities (PVGO)

- Without growth opportunities (all earnings are paid out):

$$r=\frac{DIV_1}{P_0}=\frac{EPS_1}{P_0},\ P_0=\frac{EPS_1}{r}.$$

- With growth opportunities:

$$P_0=\frac{EPS_1}{r}+NPVGO.$$

  Price of the stock equals the sum of the stock price of an equivalent firm with no growth opportunities, and the net present value (per share) of the growth opportunities.

How is the stock price affected if the firms accept negative NPV projects?

The Price-Earnings ratio

- In terms of *NPVGO*, the P-E ratio is:

$$P/E = \frac{1}{r} + \frac{NPVGO}{EPS}$$

- The higher the *NPVGO*, the higher the *P/E* ratio. High growth firms have a high P/E (*Extreme: E=0, P/E=∞!*).
- When an investor buys a stock, he/she buys both the current income (EPS) and the growth opportunities.
- The *P/E* ratio is inversely related to the discount rate *r*. High risk stocks have higher discount rates, hence lower *P/E* ratios.
- Remember, market prices *perceptions* about the future, not the future itself!

Some examples

- Fledgling Electronics has a market capitalization rate of 15%. The company is expected to pay a dividend of \$5 in the first year, and thereafter, the dividend is predicted to grow indefinitely at 10% per year.

$$P_0 = \frac{DIV_1}{r-g} = \frac{5}{0.15-0.10} = \$100.$$

Suppose EPS=\$8.33, then

$$\text{payout ratio} = \frac{DPS}{EPS} = \frac{5}{8.33} = 0.6.$$

Plowback ratio=1-0.6=0.4.

If ROE=0.25, then

$$\begin{aligned} \text{growth rate } g &= \text{plowback ratio} \times \text{ROE} \\ &= 0.4 \times 0.25 \\ &= 0.10 \end{aligned}$$

If there is no growth,

$$\text{share price} = \frac{EPS_1}{r} = \frac{8.33}{0.15} = \$55.56.$$

Therefore,

PVGO=\$100-55.56=\$44.44.

- The YTM on a bond is the interest rate you earn on your investment if interest rates don't change. If you actually sell the bond before it matures, your realized return is known as the holding period yield (HPY).

a. Suppose you buy a 10% coupon bond making annual payments today for $1,100. The bond has 10 years to maturity. What rate of return do you expect to earn on your investment?

b. Two years from now, the YTM on your bond has declined by 2.5%, and you decide to sell. What price will your bond sell for? What is the HPY on your investment? Compare this yield to the YTM when you first bought the bond. Why are they different?

- My Money Inc., just paid a dividend of \$2.50 on its stock. The growth rate in dividends is expected to be a constant 6.5% per year indefinitely. Investors require a 20% return on the stock for the first three years, then a 15% return for the next three years, and then a 10% return thereafter. What is the current share price for My Money stock?