Do Hedge Funds Have Enough Capital?

A Value at Risk Approach

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Abstract

In this paper, we examine the risk characteristics and capital adequacy of hedge funds using Value-at-Risk (VaR) as the criterion for measuring risk and estimating capital requirements. Using VaR based capital criterion, we find that a vast majority of hedge funds are adequately capitalized. The level of under-capitalization for defunct funds (7.5%) is higher than that for live funds (3.1%), indicating that capital problems may have contributed to their demise. The assumption of normality distorts and understates VaR. Hedge fund return distributions have significantly high kurtosis, indicating fat tails in the return distributions. This departure from normality makes it imperative to use empirical return distributions in estimating VaR. Further, using extreme value theory, we confirm that the VaR estimated from empirical return distributions is accurate and unbiased. These results are supported by those obtained by using daily hedge fund returns as well as S&P 500 index returns. Our study uses an extensive data set of more than 2,000 hedge funds. Hence the large extent of adequate capitalization that we find in the hedge funds industry has strong implications for fund managers, lenders, investors, and regulators.

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The hedge fund industry is one of the fastest growing sectors in finance. Due to fancy investment instruments, flexible investment strategies, sophisticated investors, limited regulatory oversight, and advantageous fee structures, hedge funds have gained tremendous popularity in the last decade. However, despite their popularity, there are very few studies in the hedge fund area, which is partly due to the non-availability of hedge fund data. Since the SEC does not regulate hedge funds, they are not required to disclose any information. Hedge funds only voluntarily report data to vendors in order to attract potential investors. Since reporting is voluntary, better-performing funds may choose not to disclose information to data vendors since they may not need to raise any more capital. This data problem in the hedge fund industry is slowly being corrected by some data vendors who have started recording even daily return data for some hedge funds.

The few studies there are, in hedge funds, analyze hedge fund performance, compare it to different market indices and benchmarks, and investigate the reasons for differences in fund performance across styles. Although these studies also examine hedge fund volatility, they do so in the traditional mean-variance framework. These studies include Fung and Hsieh (1997), who extend Sharpe's (1992) asset class factor model to include more diversified hedge fund strategies. They argue that the non-traditional and highly dynamic hedge fund investment strategy can provide an integrated framework for style analysis. In their study, they combine hedge fund data with commodity trading data. Brown, Goetzmann, and Ibbotson (1999) examine the performance of offshore hedge funds. They attribute offshore fund performance to style effects rather than managerial skills. Ackermann, McEnally, and Ravenscraft (1999) report that the comparison of hedge funds and market indexes yields mixed findings. They conclude that hedge funds outperform mutual funds. Liang (1999) documents that hedge funds dominate mutual funds in the mean-variance efficient world, with hedge fund investment strategies being dramatically different from those of mutual funds. Agarwal and Naik (2000) propose a general asset class factor model comprising of both option-based strategies and buy-and-hold strategies to benchmark hedge fund performance.

An important question, unanswered as yet, is about the risk profile of hedge funds. The debacle of the well-known hedge fund, Long-Term Capital Management LP (LTCM), only highlights the need for more academic and practitioner studies in the area of hedge fund risk exposure and capital adequacy.[1] In a recent study Jorion (2000) studies LTCM's Value-at-Risk (VaR) and estimates the amount of capital that was necessary to support its risk profile. He finds that LTCM severely underestimated its risk due to its reliance on short-term history and risk concentration. Although Jorion's study is the first one to analyze hedge fund VaR, he examines only a single fund. Also, the LTCM debacle was an extreme event, which may not be representative of the hedge fund industry as a whole. In this paper, we study the hedge fund industry as a whole. Fung and Hsieh (2000b) examine hedge fund performance and risk in some major market events/crisis. They adopt a traditional mean and standard deviation approach. In this paper, we use the VaR approach to study hedge fund risk. Lhabitant (2001) studies the investment styles of hedge funds and fund of funds, and extends the factor models to estimate the VaR of the funds, and reports preliminary VaR figures and stress testing results using his methodology. However, he does not use the return information directly in estimating the VaR, and does not examine any capital adequacy issues in this industry.

The lessons from LTCM motivate us to ask the following questions. How risky are hedge funds, in general? Do a majority of hedge funds have adequate equity capital to back their positions? Was LTCM's capital problem indicative of a more widespread capital problem in the hedge fund industry, or was it just an extreme outlier? How are hedge fund returns distributed? How should hedge fund risk be measured, if there is significant non-normality in hedge fund returns? Is VaR a better risk measure for hedge funds than traditional measure like leverage and standard deviation of returns? Is VaR a better measure for estimating hedge fund capital requirements? Are there significant differences in capitalization across hedge fund styles?

In this study, we answer the questions we have raised above. Like Jorion (2000), we propose a VaR approach, since VaR not only measures the maximum amount of assets a fund can lose in a certain time period with a certain probability, but can also be used to measure the equity capital needed to cover the losses. We analyze the VaR for each fund, its distribution across all funds, and compute a VaR based estimate of required equity capital for each fund. This required equity is then compared to the actual fund equity to determine how many hedge funds are under capitalized. We conduct various robustness checks on the VaR estimates that we obtain using the

[1] One of the reasons for LTCM's near-failure was its high leverage ratio (50:1) and the lack of adequate capital to back its portfolio of positions. Due to the Russian debt default and its impact on financial markets around the world, LTCM lost majority of its assets and faced numerous margin calls from different brokerage houses and investment banks. Not having enough equity to cover these calls, LTCM was on the brink of bankruptcy. Realizing the potentially disastrous impact it could have on global financial markets, the Federal Reserve decided to facilitate a bail out package for LTCM through other financial institutions.

empirical distribution of monthly fund returns. We first analyze some funds for which we have daily returns data, over almost one year, and evaluate their capital adequacy. Then, we use the extreme value theory to refine our estimates of VaR for the sample of funds with daily returns data, as well as for a subset of funds for which we have more than 10 years of monthly returns data, to identify potential biases, if any, in our empirical VaR estimation approach. We also compare VaR from the empirical return distribution with the standard deviation based VaR (assuming normality) to understand the error that can be introduced due to the normality assumption. We then conduct tests on daily and monthly S&P 500 index return data over a 30 year period, to address the return frequency issue in more detail, as well as to benchmark hedge fund risk to the risk of an equivalent position in a broad based equity index.

We find that a vast majority of hedge funds are adequately capitalized, contrary to the commonly perceived notion of hedge funds being too risky. We conclude that the LTCM debacle was an extreme case, not representative of the hedge fund industry as a whole. The results from our extreme value theory (EVT) estimation of VaR suggest that our empirical VaR methodology is fairly accurate, since the EVT VaR estimates do not deviate significantly from the empirical VaR estimates. We find significant non-normality in hedge fund returns, in terms of high kurtosis. Due to this reason, traditional standard deviation based risk measures cannot capture the true risk of hedge fund returns. Hence VaR estimates based on the empirically observed distribution of returns capture hedge fund risk more appropriately. Leverage does a poor job in capturing hedge fund risk. Our results are robust to time horizon and return frequency issues. In benchmarking tests, we find a majority of hedge funds to be less risky than an equivalent investment in the S&P 500 index, which further reinforces our main conclusion from the paper, that the hedge fund industry is adequately capitalized. Therefore, our study sheds light on the risk management, capital adequacy, and regulatory issues in the hedge fund industry.

This paper is organized as follows. Section 1 explains the concept of VaR and its application in determining capital requirements. Section 2 describes the data used in the study. Section 3 explains the research methodology. The empirical results are presented and discussed in section 4. Section 5 concludes.

1. Value at Risk and Capital Adequacy

VaR can be described as a summary measure of the worst loss that can happen over a target horizon with a given confidence level. In other words, VaR is a probability statement about the potential change in the value of a portfolio resulting from changes in market factors over a specified time interval. More formally, it describes the quantile of the projected distribution of gains and losses over the target horizon. If c is the selected confidence level, VaR corresponds to the $1-c$ lower tail. It has been one of the most important developments in risk management over the past few years, and is especially suited for measurement and aggregation of diverse risky positions across an entire institution using a common conceptual framework.

VaR is calculated in currency units (dollars) and is designed to cover most, but not all, of the losses that a risky business might face. Therefore, it has the intuitive interpretation of the amount of economic or equity capital that must be held to support that particular level of risky business activity. In fact, the definition of VaR is completely compatible with the role of equity as perceived by many financial institutions; while reserves or provisions are held to cover expected losses incurred in the normal course of business, equity capital is held to provide a capital cushion against any potential unexpected losses. Since all unexpected losses cannot be covered with 100% certainty, the level of this capital cushion must be determined within prudent solvency guidelines over a reasonable time horizon needed to identify and resolve problem situations.

The philosophy that economically determined VaR is the relevant measure for determining capital requirements for risky businesses is also being increasingly adopted by regulators and supervisors. The Bank for International Settlements (BIS) now allows financial institutions to use their own internal VaR estimates to determine the capital needed to support their activities. The Derivatives Policy Group (DPG) formed by the Securities and Exchange Commission (SEC) in 1994 also makes similar recommendations regarding the determination of capital adequacy. Therefore, the use of a VaR measure to study capital adequacy is extremely relevant and is in line with the norms and guidelines in place for other financial institutions, including banks, broker-dealers, etc.

There are three main decision variables that are required in order to estimate VaR - the confidence level, a target horizon, and an estimation model. If the objective of estimating VaR is to use it to estimate risk capital requirements, it should be chosen to be high enough so that there is very little probability that actual losses would exceed the VaR amount, since the costs of actual losses exceeding the VaR are usually very high. The confidence level should be such that the probability of bankruptcy is sufficiently low. The target horizon is related to the liquidity of the positions in the portfolio. It should reflect the amount of time necessary to take corrective action, should something go wrong and high losses occur. It should correspond to the time necessary to raise additional funds to cover abnormal losses. The VaR estimation model would, of course, determine the accuracy of the VaR calculation. As one can see, there is considerable uncertainty in choosing these variables, and often the choice is arbitrary.

The Basel Committee on Banking Supervision (1996) has set forth guidelines for commercial banks to estimate their equity capital requirement. They stipulate a 99% confidence level and a ten-day horizon. Further, to account for model misspecification and other inaccuracies in the estimation procedures, they require the resulting VaR to be multiplied by 3 to arrive at the capital requirement. The safety multiple of 3 is used to take care of estimation biases and model misspecification in VaR estimation.[2]

In this paper, we use 3 times the 99% 1-month VaR as the required equity for hedge funds. The time horizon used is 1-month instead of ten days because hedge funds are quite different from commercial banks. As pointed out by Jorion (2000), commercial banks are closely supervised by regulators, hence they are in a position to react to potential difficulties much sooner. Hedge funds are far less regulated and since they are not allowed to raise funds from the public, they would normally have a much harder time raising additional capital when needed, which would be precisely when they have suffered abnormal losses. Hence a 1-month target horizon is more appropriate for hedge funds.

2. Data

In this paper, we mainly use the hedge fund dataset from TASS Management Limited (hereafter, TASS), which contains data on 2,016 hedge funds, including 1,407 survived and 609 dissolved funds, as of July 1999. TASS reports returns on a monthly basis. The total assets under management are about $175 billion, making it one of the largest hedge fund databases for

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[2] In fact, as Stahl (1997) shows using Chebyshev's inequality, a maximum correction factor of 3 takes care of all error introduced due to misspecification of the true distribution of returns.

academic research. Liang (2000) indicates that the TASS data has some advantages over the other databases because it contains more dissolved funds and is more accurate in fund characteristics.

In order to estimate the empirical distribution of fund returns, we need a certain minimum number of observations. We use a three-year window as the minimum time period to estimate the return distribution.[3] Funds with less than three-year return history are deleted. In addition, we need fund equity information to evaluate whether a fund is under-capitalized, and fund style classification in order to analyze funds on a style basis. After all these criteria are imposed, we have 860 funds left in our study, out of which there are 820 live funds and 40 dead funds. The three-year window for live funds is chosen to be from August 1996 to July 1999, so that the most recent information about funds can be analyzed.[4] For dead funds, we choose the return window to be the three-year period from October 1995 to September 1998, so that we can analyze all funds that dissolved from October 1998 to July 1999. This is done in order to include funds that dissolved as a consequence of the Russian debt crisis and the LTCM debacle in August-September 1998. Note that the return period that we analyze was very volatile, since it covers the Asian currency crisis of 1997 as well as the Russian debt crisis of 1998.

Our data is categorized by fund styles, as defined by TASS. These styles are US equity hedge, European equity hedge, Asian equity hedge, global equity hedge, dedicated short seller, fixed income directional, convertible fund, event driven, non directional/relative value, global macro, global opportunity, natural resources, pure leveraged currency, pure managed future, pure emerging market, pure property, and fund of funds.[5] This style classification allows us to study hedge fund risk and capital adequacy by investment styles.

In addition to the TASS data, we also use daily return data from Hedge Fund Research, Inc. (hereafter, HFR) as well as daily and monthly data for the S&P 500 index for comparison purposes. We have daily return information and other fund characteristics for 33 hedge funds, as of December 31, 2000. After deleting two funds with no fund equity information, we have 31 funds left in the sample. These fund returns are from January 24, 2000 to December 27, 2000, with

[3] For robustness, we also tried a two-year window and a five-year window. The results were very similar, therefore we report our results from the three-year window only.

[4] We use the same time period for all hedge funds so that the results across funds are comparable.

[5] These 17 styles are overlapping, which indicates that a fund could belong to more than one style category. However, under this style definition, all funds are coded as either live or dead funds. There is no ambiguity. Although there is another set of style definition in the TASS data, many funds are not classified as either live or dead funds. Since a fund's disappearance is important, we use the first style definition instead of the second.

27 out of the 31 funds having more than 200 daily return observations, the minimum number of observations being 119.

The S&P 500 return information is from January 1970 to July 1999, with 7475 daily or 355 monthly observations. To match with the hedge fund information, we also analyze a three-year window from August 1996 to July 1999 for the S&P 500 index in addition to the 355-month window.

3. Research Methodology

Hedge funds often have complex portfolios including nonlinear assets like options, interest rate derivatives, etc. For such a portfolio, estimating the VaR (or equivalently the probability of large downward moves, deep in the tails of the probability distribution) is a difficult problem, since both the non-Gaussian nature of the fluctuations of the underlying assets and the non-linear dependence of the price of the derivatives must be dealt with. Moreover, there is no data available on the position holdings of hedge funds, since it constitutes proprietary trading information that they never disclose. Therefore, it is not possible to estimate the VaR of hedge funds by doing a position level analysis. The best data we have is data on monthly returns reported by the hedge funds, which we use to estimate the VaR as accurately as possible.[6] Note that almost all major data vendors provide return information either on a monthly basis or on an annual basis. For example, TASS (used by Fung and Hsieh (1997) and Liang (2000)), Hedge Fund Research Inc. (used by Ackermann, McEnally, and Ravenscraft (1999), and Agarwal and Naik (2000)), and Managed Account Reports, Inc. (used by Ackermann, McEnally, and Ravenscraft (1999)) report monthly returns while the U.S. Offshore Funds Directory (used by Brown, Goetzmann, and Ibbotson (1999)) reports annual returns. Hedge Fund Research, Inc. has recently started collecting daily return information for a few dozen hedge funds; we use these daily returns to cross validate our main results obtained from monthly data.

For leverage information, TASS reports two numbers - the average leverage ratio and the maximum leverage ratio. The average leverage refers to the most recent leverage used by the fund, averaged across all its positions. The maximum leverage refers to the maximum leverage that the fund is allowed to use. Hence, the average leverage provides a measure of what the fund is doing right now, while the maximum leverage indicates what the fund could potentially do, in

[6] We use the extreme value theory to improve the accuracy of this VaR approach, and do robustness checks to make inferences about the accuracy of our empirically estimated VaR.

terms of taking on debt and exposing the investors to capital risks. To save space, we report the average leverage number only in this paper.

3.1 VaR from empirical distribution of returns

We use monthly return data to estimate an empirical distribution of returns for each hedge fund separately. As stated earlier, we impose a requirement that at least 3 years of return data (36 monthly return observations) be available for the fund to be included in the study, in order to reliably estimate the distribution. We use this empirical return distribution, along with the net asset value (equity), to estimate the VaR.

Our empirical VaR is estimated as follows:

$$VaR = \left(R_{mean} - R_{99\%}\right) \times NAV \tag{1}$$

where

VaR = 99% 1-month VaR,

R_{mean} = Mean fund return from historical distribution,

$R_{99\%}$ = Cut off return at 99% confidence level from the empirical distribution on the left tail,

NAV = The total net asset value (equity) of a fund.

As can be seen, this VaR is a VaR relative to the mean, which specifies the dollar loss relative to the expected return, instead of the VaR from zero returns, which is the dollar loss relative to zero (without reference to the expected return over the target horizon). For a horizon of a month, hedge fund returns can be quite significant. Hence, VaR relative to the mean is more appropriate since it views risk in terms of the deviation from the expected value on a target date, correctly accounting for the time value of money.[7] The capital requirement is then taken as 3 times this VaR number, which is the minimum amount of equity necessary to support the level of risk inherent in each fund's operations. As explained earlier, the multiplier of 3 takes model misspecification and estimation errors into account and is recommended by the Basel Committee on Banking Supervision.

[7] We computed the VaR relative to zero returns as well, in order to check the robustness of our conclusions, and the results were very similar.

One may argue that the three-year minimum return history requirement to estimate hedge fund VaRs may introduce a survivorship bias in our VaR estimation. By ignoring funds with less than a three-year history, we throw away younger (and potentially riskier) funds. Therefore, we may underestimate the true degree of under-capitalization. However, we consider dead funds as well in our study, not just funds that have survived, hence the survivorship bias is mitigated. If we had included all the funds in our sample, our VaR estimates could have been higher for some funds, and we may have detected a higher degree of under-capitalization in the funds. To obtain reliable VaR estimates, we need a certain minimum number of observations, hence data constraints prevent us from analyzing all funds. Nevertheless, we do analyze a large number (860) hedge funds, and, as we show later in the paper, our results are very robust to return frequency, time period and estimation methodology issues.

3.2 The degree of under-capitalization

The risk capital requirement estimated using VaR is compared with the actual equity capital backing the positions of these hedge funds, in order to evaluate the capital adequacy of the funds. We compute an under-capitalization ratio (*U-cap* ratio) defined as follows,

$$U - cap = \frac{E_{actual} - E_{required}}{E_{required}} \tag{2}$$

where the required equity is 3 times the 99% 1-month VaR of the fund, while the actual equity is taken from the data. A *U-cap* ratio less than zero implies that the actual equity is not sufficient to cover the risk of the portfolio as per the VaR approach. Hence funds with *U-cap* ratios less than zero are identified as being under capitalized. Further segmentation by fund styles, and by their current status (whether they are living or dead), helps us evaluate the risk exposures of these funds by styles, and allows us to determine the impact of capital adequacy on the survival rates of hedge funds. In addition, we compare the hedge fund *U-cap* ratios to that from an equivalent investment in the S&P 500 index, in order to evaluate the risk of hedge funds relative to a broad equity market index.

3.3 VaR using Extreme Value Theory (EVT)

The objective of estimating VaR in this study is to estimate the capital requirement for hedge funds. Equity capital, by definition, is the capital reserve required to bear unexpected losses. Most of the unexpected losses arise due to extreme events in financial markets, such as the Russian debt default and the LTCM crisis in September 1998, the breakdown of the European Monetary system in September 1992, the stock market crash in October 1987, etc. These extreme events result in most of the losses suffered by a fund or a financial institution, which the required equity capital is supposed to cover. Therefore, the estimation of capital requirements can be considered to be an extreme value problem. While estimating VaR, we are concerned with the behavior of the return distribution deep into the left tail, which corresponds to rare phenomena outside of the range of normal observations. Extreme Value Theory provides a firm theoretical foundation to model such extreme events, and can be used to estimate tail related risk, and hence VaR.

EVT considers the statistical distribution of extreme returns, instead of the distribution of all returns. It provides us the limiting distribution of the extreme returns over a time period, which is independent of the distribution of returns itself. EVT focuses only on extreme values rather than all the data, hence fitting only the tail of the distribution. Therefore, EVT can capture event risk which normal distributions cannot. For modeling extreme values, there are two different, but related, approaches. The first approach consists of fitting one of the three standard extreme value distributions (Fréchet, Weibull, or Gumbel) to block maxima values in a time series, while the second approach models the distribution of exceedances over a threshold as a generalized Pareto distribution (also known as Peak Over Threshold (POT) method).

Let R_i be the (random) returns for a hedge fund, at any frequency, say daily intervals. In the first approach (the block maxima method), let X_i be the maximum (or minimum) return values observed in successive but non-overlapping periods. The X_i observations constitute extreme returns over the time period. If the return observations are statistically independent and drawn from the same distribution, then the exact cumulative distribution of the maximum ($H(x)$) can be written as a function of the cumulative distribution of all returns ($F(x)$), and the length of the period, n, as follows:

$$H(x) = [F(x)]^n \tag{3}$$

However, if the distribution of returns is not known, then this result is of little practical use. In this case, the Fisher-Tippett (1928) theorem shows that in the limit, the asymptotic distribution of the maximum variable X_i, reduced by a location parameter μ and a scale parameter σ, converges to a generalized extreme value (GEV) distribution as follows:

$$\frac{X_i - \mu}{\sigma} \xrightarrow{d} H, \qquad where$$
$$H_\xi(x) = \begin{cases} \exp\left[-(1+\xi x)^{-1/\xi}\right] & if \quad \xi \neq 0, \ 1+\xi x > 0 \\ \exp\left[-e^{-x}\right] & if \quad \xi = 0 \end{cases} \tag{4}$$

The parameter ξ is called the tail index, since it models the tail of the distribution. The scale parameter (σ) can be interpreted to be the volatility of the extreme values, while the location parameter (μ) represents the average of the extreme values. According to the tail index value, three types of extreme value distributions are obtained: the Fréchet distribution (ξ>0, fat tails), the Weibull distribution (ξ<0, thin tails), and the Gumbel distribution (ξ=0, no tails). Most financial time series usually exhibit fat tails, and hence have ξ>0.

In the POT method, the distribution of the exceedances over a threshold is modeled. For random returns R_i, the POT method models the distribution F_u of values of returns above a certain threshold u. This conditional excess distribution function is defined as

$$F_u(y) = P\left(R - u \leq y \middle| R > u\right), \qquad where \quad y = r - u \tag{5}$$

As observed by Pickands (1975), the conditional excess distribution function, for large u, is well approximated by the generalized pareto distribution (GPD) as follows:

$$F_u(y) \approx G_{\xi\sigma}(y), \qquad u \to \infty,$$
$$G_{\xi\sigma}(y) = \begin{cases} 1 - \left(1 + \dfrac{\xi}{\sigma} y\right)^{-1/\xi} & if \quad \xi \neq 0 \\ 1 - \exp\left(-y/\sigma\right) & if \quad \xi = 0 \end{cases} \tag{6}$$

In this case as well, ξ is the tail index and σ is the scale parameter. This generalized distribution nests three standard distributions: the pareto distribution (ξ>0, polynomially decreasing tail), the uniform distribution (ξ<0, short tail), and the exponential distribution (ξ=0, exponentially decreasing tail). Again, only distributions with ξ>0 are suitable for modeling financial time series which exhibit fat tails.

Between these two methods of implementing EVT, the threshold method is preferred in applications where there is less data available, since this method used data more efficiently. Hence, in this paper, we use the POT method to estimate the VaR.

In the POT method, the conditional excess distribution function (F_u) can be written in terms of the cumulative distribution function (F) of all returns as follows:

$$F_u(y) = \frac{F(u+y)-F(u)}{1-F(u)} = \frac{F(x)-F(u)}{1-F(u)}, \qquad where \quad y = x-u$$
$$\Rightarrow F(x) = [1-F(u)]F_u(x) + F(u) \tag{7}$$

From this, an analytical expression can be derived for the VaR. F_u can be replaced by the generalized pareto distribution. F(u) is deterministic, since it is the fraction of observations below the threshold, hence F(u)=(N-n)/N, where n is the number of observations above the threshold, while N is the total number of observations. Therefore,

$$F(x) = \left[\frac{n}{N}\right]\left[1-\left(1+\frac{\xi}{\sigma}(x-\mu)\right)^{-1/\xi}\right] + \left(1-\frac{n}{N}\right), \qquad for \quad \xi \neq 0$$
$$= 1-\frac{n}{N}\left(1+\frac{\xi}{\sigma}(x-\mu)\right)^{-1/\xi} \tag{8}$$

F(x) denotes the VaR confidence level. Let p denote the probability of exceeding the VaR, which would be given by 1-F(x). For example, for the 99% VaR, p would be 0.01. Also, the x itself denotes the VaR. Substituting for F(x) and x, we get the expression for VaR as

$$VaR_p = u + \frac{\sigma}{\xi}\left[\left(\frac{N}{n}p\right)^{-\xi} - 1\right], \qquad for \quad \xi \neq 0 \tag{9}$$

When ξ=0, the form of the generalized pareto distribution is different, and the expression for VaR is

$$VaR_p = u + \sigma \log\left(\frac{n}{N} p\right), \qquad for \quad \xi = 0 \tag{10}$$

For a sample of return observations, the parameters of the generalized pareto distribution can be estimated using maximum likelihood methods.[8] Using the estimated parameters, appropriate VaR estimates can be obtained at any confidence level based on (9) and (10).

In this paper, we estimate the 99% EVT based VaR for the set of funds for which we have daily return observations from HFR. We also estimate the EVT VaR for the subset of funds (105 out of the 820 live funds) for which we have more than 10 years of monthly observations from TASS. To reliably estimate any extreme value distribution, we need a large number of observations. Therefore, data constraints allow us to estimate the EVT VaR only for either the daily return data, where we typically have more than 200 return observations for each fund, or for funds with a long time series of monthly returns. The primary objective of estimating the EVT based VaR is to compare it to the empirically estimated VaR, and identify potential biases, if any, in our results.

3.4 VaR using normality assumption

Many traditional risk based capital measures assume the return distribution to be normal, though often there are significant departures from normality (our results show a significantly high level of kurtosis and skewness in hedge fund returns across fund styles). A comparison of risk capital measures based on the empirical distributions with those based on the assumption of normality would highlight the error introduced in estimating capital requirements when returns are assumed to be normal. Towards this objective, we re-estimate the VaR of each fund using the standard deviation of historical returns, assuming the return distribution to be normal, instead of directly estimating it from the empirical distribution as in the previous case. Hence, in this case, the VaR is defined as

[8] See Bali (2001), Longin (2000), Kellezi and Gilli (2000), Smith (1987) and others for details of the estimation procedure.

$$VaR = \left[\left(\sigma_R \times 2.326\right) \times NAV\right] \tag{11}$$

where σ_R is the standard deviation of historical fund returns. The capital requirement is again specified as three times this VaR, and the under-capitalization ratio (*U-cap* ratio) is computed in a similar manner. Again, a *U-cap* ratio less than zero would imply a level of equity less than that required, and hence the fund would be categorized as being under capitalized. In this section as well, the funds are segmented by investment styles and by their current status (whether they are living or dead), in order to evaluate the risk exposures of funds by styles and the impact of capital adequacy on their survival rates. The differences in the levels of under-capitalization using the VaR from the empirical distribution and the standard deviation based VaR can be attributed solely to the departures from normality in the actual return distributions of hedge funds.

3.5 Robustness tests and benchmarking to S&P 500 index

One may argue that the results based on monthly returns may smooth out the true volatility of hedge fund returns at the aggregate level, hence underestimate the true VaR and capital adequacy for the industry. In addition, previous studies have found that daily stock returns exhibit more non-normality than monthly returns. If hedge fund returns are significantly non-normal, then it should be easier to detect this deviation from normality using daily data than using monthly data. Therefore, daily returns may be more appropriate than monthly returns for estimating the risk exposure for the hedge fund industry. In this paper, we use 31 hedge funds with daily returns from HFR to cross validate our results obtained from monthly information.

In addition, we use both the S&P 500 daily and monthly returns to benchmark hedge fund risk and to study the return frequency issue in more detail. The objectives of using the S&P 500 return data are fourfold. First, we want to address the issue of potential bias that may be introduced by using monthly returns instead of daily returns. To understand the impact of return frequency on estimated VaRs, we use the S&P 500 data from January 1970 to July 1999. We estimate the monthly VaR, first using the 355 monthly return observations, and then using the 7,475 daily return observations over the exact same period. A comparison of the two VaRs would highlight the differences caused by return frequency alone. Second, we want to understand the potential bias that may be introduced in our results due to the use of the shorter three-year window of returns for hedge funds. To understand this bias, we compare the monthly VaR computed using the 355 monthly observations over a 30 year period to the monthly VaR estimated from only 36 months of S&P returns, over the exact time period that we use for the hedge funds, i.e., from August 1996 to July 1999. This comparison indicates how different the risk estimates might be if a longer time history of returns were used. Third, we compare the empirical S&P VaR To the EVT based VaR for both monthly and daily S&P returns, to identify potential biases in our empirical VaR estimation methodology. Using daily S&P return observations, we also report the EVT VaR estimates using different number of extreme observations, to test the robustness of our EVT estimation methodology. Fourth, we use the S&P 500 risk estimates to benchmark hedge fund risk. Our objective is to quantitatively assess, for the entire hedge fund industry, the risk of hedge funds when compared to a broad equity market index. Specifically, we wish to understand how the risk profile of each hedge fund compares to average market risk.

4. Results

4.1 The capital adequacy of hedge funds

Table 1 presents the distribution statistics of hedge fund returns by investment styles. All the numbers in the table are the medians across all funds in the same fund style. Several inferences can be drawn from this table. Firstly, live funds outperform dead funds in most styles. The median live fund earns an average monthly return of 1.02%, compared with the 0.56% return for a median dead fund. Dead funds are slightly more volatile than the live ones, measured by the standard deviation of returns. These results are consistent with Fung and Hsieh (2000a) and Liang (2000), who document that funds die mainly due to poor performance. Secondly, hedge fund returns do not exhibit a high level of skewness (compared with a traditional cutoff point of 0.3), for both live and dead funds, but they do have very high kurtosis, which makes hedge fund return distributions significantly non-normal. All investment styles show high kurtosis above three for live funds, which indicates that hedge fund returns have fat tails and more extreme return values, because of the dynamic and risky strategies they employ. For example, event driven funds have a median kurtosis of 6.74 for live funds and 9.86 for dead funds. This is consistent with the notion that hedge fund risk is more event-driven and non-linear than regular price fluctuations under normal circumstances. Often, hedge funds take opportunistic trading strategies and bet on major markets events worldwide. Hedge fund returns are heavily affected by these events; hence extreme positive (as in the famous “attach” on Sterling by George Soros’ funds in 1992) and negative (as in the market downturn for LTCM in 1998) returns may be realized. All these events may cause some observed returns to be well beyond three times the standard deviation, which would have virtually zero probability of occurrence under the normal distribution assumption. Therefore, using just the second moment to measure hedge fund risk is inappropriate, and we turn to VaR for evaluating hedge fund risk in this paper. Table 1 also shows that there are significant differences in hedge fund return distributions across investment styles, implying that a study by fund styles is more meaningful than an aggregated study that groups all hedge funds together.

Table 2 presents statistics on the medians of the fund sizes and absolute VaR numbers across fund styles for both live and dead funds, in order to understand the magnitude of the dollar values in question. In the live funds group, the average fund size ranges from $1.03 billion for the global macro style to only $2.5 million for the natural resources style as of July 1999. Because of the differences in fund size across investment styles, the average estimated VaR ranges from $148.8 million for the global macro funds to only $0.5 million for the natural resources funds. The 99% VaR shows the loss limit that is expected to be breached 1% of the time. For example, on average, US equity hedge funds can lose more than $13.6 million in a month with a 1% probability. It is not surprising to find that dead funds are generally smaller than live funds. Dead funds lose capital because of poor performance, or they are unable to reach a critical mass, so they die. The months immediately preceding a funds demise are usually characterized by large negative returns, hence these funds shrink in size considerably before closing down. Because fund assets differ, a VaR relative to fund assets is more appropriate than the absolute VaR for a comparison purpose. When analyzing the VaR as a percentage of fund size, we find that generally, dead funds have higher relative VaRs than live funds, which reflects the higher risk implicit in dead funds. For example, the median (mean) relative VaR is 8.4% (8.9%) for the live funds, compared with a higher 11.5% (11%) relative VaR for the dead funds. Across styles, the Asian equity, Global equity, Global opportunity, and emerging market funds are particularly riskier than the other styles for both live and dead funds.

The main results of this paper are presented in Table 3. The numbers reported are the total number of funds, the number of funds that are under-capitalized, the percentage of under-capitalized funds, the level of under-capitalization (*U-cap* ratio) and average leverage. The results are reported for both live and dead funds under each style classification. The under-capitalization figures in Table 3 have been calculated using the VaRs based on the empirical distribution of monthly returns.

It is surprising to find that very few hedge funds are under-capitalized, for both the live and dead fund groups. For the live funds, about 3.1% of the funds are under-capitalized, while the corresponding fraction is 7.5% for the dead funds. The median (mean) under-capitalization ratio is 2.5 (4.6) for live funds, compared with 2.2 (3.2) for dead funds. Remember that the lower the under-capitalization ratio, the less the actual equity capital is, as compared to the required equity. Comparing the two fund groups, we observe that, on average, dead funds have a greater extent of under-capitalization than the live funds. This supports the hypothesis that one of the reasons for a fund to die is under-capitalization. When a fund does not have enough equity capital, it can run into debt trouble after a market crisis occurs. Investor withdrawals and margin calls from lenders may eventually drive the fund out of business.

For the live funds, the only styles with significant levels of under-capitalization are the pure emerging market funds where 16 out of 72 funds (22.2%) are found to lack adequate capital and dedicated short sellers where 1 out 7 funds (14.3%) is under capitalized. Most of the other styles, even riskier fund styles with very high kurtosis in fund returns, like fixed income directional, event driven, and relative value funds, have not even a single fund that is undercapitalized. The same trend is observed in the dead funds group, where the only fund styles to have significant levels of under-capitalization are the emerging markets fund group with 2 out of 4 funds undercapitalized and natural resource funds with 1 out of 3 funds undercapitalized. Fund styles with significantly non-normal returns are also found to have adequate equity capital backing their positions.

Table 3 also presents the median average leverage figures for funds in each investment style. It can be seen that the relationship between leverage and under-capitalization is very weak. In fact, the correlation coefficient between the under-capitalization ratio and average leverage is 0.15 for live funds, which shows that leverage is a poor proxy for under-capitalization. A significantly negative correlation coefficient would have implied that leverage could have been used to proxy capital adequacy, but that is not the case here. For dead funds, the correlation coefficient is only -0.01, which is not significantly different from zero, indicating that there is virtually no relationship between leverage and capitalization for dead funds. This further reinforces the need to have capital estimation procedures based on VaR, instead of leverage.

These results indicate that the vast majority of hedge funds have adequate capital, even when VaR is used to estimate capital requirements. However, as we show later, traditional methods of capital estimation may not be able to detect even this level of under-capitalization.

4.2 Impact of return frequency on hedge fund risk estimation

Table 4 presents the descriptive statistics for 31 hedge funds using daily return data, 27 of which have at least 200 return observations.[9] It can be seen that these returns exhibit significant fat tails. The median kurtosis across all funds is 7.95 (as compared to 4.12 using monthly data), while it is as high as 24.41 for a convertible arbitrage fund. As with the monthly data, we do not find significant skewness in daily returns, indicating that extreme returns observations are found in both directions - upside as well as downside.

Table 4 also presents the under-capitalization ratios based on the empirical VaR estimated using the daily return data. We find just one (or 3.2%, compared with 3.1% using monthly data) hedge fund to be under capitalized, out of a total of 31 funds, which is consistent with the extent of under capitalization we had observed using monthly return data. The median U-cap ratio using daily returns is 2.8, as compared to a median U-cap ratio of 2.5 using monthly returns. However, these results must be interpreted with caution, since the funds in the daily sample are different from those in the monthly sample. Also, the data periods are different. The daily return data is during the year 2000, when the average volatility in the market was lower than that observed during the preceding three years for which we use monthly data. In spite of the differences, it is clear that a vast majority of hedge funds have adequate equity capital, irrespective of the methodology or return frequency used to estimate their risk capital.

4.3 VaR based on extreme value theory (EVT)

As discussed earlier in the paper, EVT offers an alternative approach for estimating the VaR for a fund. In fact, since EVT focuses on extreme return observations, one could argue that it may estimate the VaR more accurately, as compared to that estimated using the empirical distribution of returns. Our major results in this paper are based on the empirical VaR estimated using 36 monthly return observations, which are not enough to reliably estimate an extreme value

[9] Note that the style classification for daily return data by HFR is different from that for the monthly data from TASS, since they are provided by a different data vendor who has different style categories.

distribution. However, we can estimate the EVT VaR for the hedge funds for which we have daily return data, as well as for 105 out of the 820 live funds, for which we have more than 120 monthly returns. In both these cases, since we have enough observations, we can compare the EVT based VaR to the empirical VaR, and understand the biases, if any, in the empirically estimated VaRs. Note that EVT based VaR is estimated from equations (9) and (10).

Table 5 presents a comparison of the EVT based VaR to the empirical VaR for the 31 hedge funds with daily return data. By definition, the extreme value distribution must be estimated using extreme return observations. We report results for three alternative calibrations of the extreme value distribution, using 5%, 7.5% and 10% of the (lowest) return numbers as the sample of extreme returns in the estimation process. There is a trade-off between estimation accuracy and theoretical consistency in using more or less number of observations. If more observations are used, the maximum likelihood estimates of the extreme value distribution parameters have lower standard errors, hence the VaR estimate is more accurate. However, the theory demands that only extreme return observations be used in the estimation procedure, hence too many observations cannot be justified as being extreme observations. Therefore, for robustness, we estimate the EVT VaR using varying number of observations (5%-10% of the total return numbers), to make sure that our EVT VaR estimates are not unstable.

From table 5, one can arrive at two main inferences. First, the estimation of the EVT VaR is very stable, and does not depend critically on the number of observations used, within the 5% - 10% range. The median ratio of EVT VaR to empirical VaR for the same fund is 1.08 using 5% of the observations, and 1.01 using 7.5% or 10% of the observations. For many of the funds, the differences are statistically insignificant. Second, the EVT VaR appears to be only marginally higher than the empirical VaR, on average. The maximum differences are found for one distressed securities fund, where the EVT VaR is 25% higher than the empirical VaR. For most of the other funds, the EVT VaR is very close to the empirical VaR. For example, for the first fund (an equity hedge fund), the 99% 1-month empirical VaR is $10.4 million, while the corresponding EVT VaR based on 5% of the 216 daily return observations is $11.9 million, which is only 14% higher. Note that the equity requirement for the funds was assumed to be 3 times the empirical VaR. As shown by these EVT results, this multiplier of 3 more than compensates for any error in the empirical VaR estimation.

Figure 1 presents box and whiskers plots of the ratios of the EVT based VaR to the empirical VaR for the 105 hedge funds for which we have more than 10 years of monthly return data, using three different definitions for extreme return observations - the lowest 5%, 7.5% and 10% of returns. Again, it can be seen that the median ratios for these funds is just above 1, with very few funds having EVT based VaRs more than 20% higher than the corresponding empirical VaRs. In this case as well, the multiplier of 3 used for estimating equity requirements more than compensates for even the most extreme cases where the EVT based VaRs are close to 50% higher than the empirical VaRs.

In both the cases above, the results based on EVT estimation show that using the empirical VaR for capital estimation does not introduce any systematic biases.

4.4 VaR based on standard deviation

In the previous sections, we report results from VaR estimated using the empirical distribution of historical fund returns, where we do not make any assumptions about the underlying return distribution. We also report in Table 1 that hedge fund returns have significant departures from normality, especially a high level of kurtosis (indicating fat tails in the distribution). An interesting issue then is to compare the VaR from the empirical distribution with that based on the assumption of normality, using the standard deviation of historical returns. By this analysis, we can understand how much error is introduced by imposing the constraint of normal distribution. Hence, we re-estimate the VaR for each fund using the standard deviation of historical returns and calculate the degree of under-capitalization. The estimation procedure is explained in section 4 and equation (11).

Table 6 reports the number of funds and the percentage of funds that are under-capitalized based on the estimated VaR assuming normality. Comparing the results of Table 6 with those of Table 3, we find that using the standard deviation based VaR leads to an underestimation of capital requirements, especially for live funds. Standard deviation based VaR is able to detect under-capitalization in only 1.8% of the live funds, and in only 2.5% of the dead funds, while the corresponding numbers are 3.1% and 7.5% respectively using the VaR estimated from empirical distributions, where we make no assumptions on the return distribution. This is not surprising because assuming normality ignores the fat tails of hedge fund distribution, which in turn underestimates the risk of extremely low return realizations, and hence underestimates the extent of under-capitalization.

Therefore, we conclude that estimating the VaR (and hence the capital requirement) by assuming a normal distribution, instead of using the empirical return distribution, significantly understates the probability of realization of extremely low returns. It does not capture extreme returns or fat tails in return distribution. This could result in funds keeping a lower capital cushion than what is dictated by their risk profiles, thereby leading to higher chances of failures due to bankruptcy.

4.5 Benchmarking hedge fund risk to S&P 500 index

Table 7 presents the 1-month 99% VaR estimates for a $100 million position in the S&P 500 index, using monthly and daily returns over the 30-year period from January 1970 to July 1999, and using monthly returns over the 3-year period from August 1996 to July 1999. The table also presents descriptive statistics for the return distribution for the three cases. Daily S&P returns exhibit significantly higher kurtosis than monthly returns over the same period. The negative skewness in the returns is also more pronounced for daily returns than for monthly returns. However, the estimates of VaR are not very different - the VaR using daily returns is $12.6m, while it is $11.5m using monthly returns (about 9% lower than that using daily returns). The bias introduced in the VaR estimation by using monthly returns appears to be small.[10] The same is true for the VaR estimated for the shorter, three-year period, for which the VaR for the S&P index is $13.6m. Since the three-year period was a period of high volatility, the computed VaR should be higher than that estimated over a longer time horizon. Once again, the differences in the three VaR estimates are small, which indicates that the VaR estimated for hedge funds using monthly data over a three-year period is fairly accurate, and the return frequency and data period issues do not introduce any significant bias in the results.

As a further check on the robustness of our extreme value theory estimation procedure, table 7 presents the EVT VaRs for the S&P 500 index using 5% of the lowest return observations as extreme observations. These VaR figures are very close to those obtained using the empirical return distributions, for monthly as well as daily S&P returns.[11] Again, the empirical VaR estimates are unbiased, and the EVT VaR estimates do not appear to be very sensitive to the number of extreme return observations used in the estimation, within a reasonable range.

[10] Note that we use 3 times the 99% 1-month VaR as the estimate of capital requirement, hence the multiplier of 3 adequately covers any biases that may be introduced by using monthly returns.

[11] We also estimated the EVT VaRs for the S&P 500 index using different number of extreme return observations, ranging from 0.5% to 10% of the total observations (using the 7475 daily returns). The EVT VaR estimates were very similar in all those tests.

The relative VaR for the S&P index using monthly returns over three years is 13.6%. In contrast, the median relative VaR for the 820 live funds in our sample is 8.9%, which indicates that the risk of most funds is less than or comparable to the risk of an equivalent position in the S&P 500, which is a broad based US stock market index. Only 5 styles have median relative VaRs greater than 13.6% (global equity, Asian equity, global opportunity, natural resources, and emerging market). Interestingly, four out of these five styles invest in foreign securities, which are more volatile than the US securities. Therefore, contrary to what might be expected, most hedge funds appear to maintain fairly low risk profiles, with adequate capital backing their positions. On average, they are less risky than a broad equity market index.

5. Conclusion

In this paper, we study the risk characteristics and capital adequacy of hedge funds, using Value-at-Risk (VaR) as the criterion for measuring risk and estimating capital requirements. Our study is extensive, as we use a large hedge fund database and analyze a total of 860 funds, including live and dead, across all investment styles. In addition to the monthly return information from these funds, we use daily return data for 31 funds to cross validate our results. We also compare our hedge fund results with those from the S&P 500 index.

VaR makes a probability statement about the potential change in the value of a portfolio resulting from changes in market factors over a specified time horizon. Since it is designed to cover most of the unexpected losses in a risky business, it has the intuitive characterization of the amount of economic or equity capital that must be held in order to support that level of risky business activity. Therefore, a VaR based measure naturally lends itself to being applied for estimating the equity capital requirement of an institution. Indeed, regulators prescribe that VaR based measures be used for determining capital adequacy in the banking and securities industries. In this paper, we apply this concept to the hedge fund industry, a vast sector that has remained largely unregulated till now, but some recent debacles have challenged the need for more regulation in this industry.

Using VaR based criteria, we find that the vast majority hedge funds are adequately capitalized, with only a very small proportion of them being under-capitalized. This is in strong contrast to the case of LTCM, where an abnormal 50:1 leverage ratio was employed and the fund capital was at a significant risk. We find that the level of under-capitalization using VaR based criterion is greater for dead funds (7.5%) than that for live funds (3.1%), confirming the hypothesis that a significant reason for hedge fund death is under capitalization. We also find significant differences in capitalization of funds across investment styles, indicating that some styles are more adequately capitalized than others.

For robustness, we further check our monthly results by using daily hedge fund returns. The daily return results are very consistent with the monthly return results, confirming that our finding is valid and monthly returns contain enough information to estimate hedge fund risk and capital adequacy reliably. We further compare our hedge fund results with those using the S&P 500 index returns. We find that return frequency and time periods do not significantly bias the estimates of hedge fund risk. In general, except for styles involving international investment, we do not find evidence that hedge funds are any riskier than an equivalent position in a broad based market index like the S&P 500.

In terms of actually implementing our VaR based capital approach, we find that using the empirical distribution of fund returns results in more accurate capital estimation, as compared to the alternative of assuming the fund return distribution to be normal. An analysis of fund return distribution reveals significant levels of non-normality in terms of high kurtosis, indicating fat tails in the return distribution. Therefore, estimating VaR by assuming a normal distribution severely underestimates the risk of extreme negative movements in the returns, thereby understating the true capital requirement. The accuracy of the empirical VaR estimation approach is confirmed by comparing it to the VaR estimated using extreme value theory. We find that the EVT based VaR estimates are comparable to the empirical VaR estimates, for the 31 funds with daily returns as well as for 105 funds that have more than 10 years of monthly returns data. Therefore, using the empirical distribution of returns does not introduce any bias in our VaR estimation.

Our study is the first one that extensively analyzes hedge fund risks and relates them to capital adequacy, and uses VaR instead of traditional risk measures. It has fundamental implications about risk adjusted performance measurement in hedge funds as well. Our study is also the first one to employ daily hedge fund returns. Previous studies all use monthly or annual returns. The large extent of adequate capitalization in hedge funds that we find in this paper has strong implications for fund managers, institutional lenders, investors, and financial regulators.

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Table 1

Descriptive Statistics for Hedge Funds

This table presents the descriptive statistics for the return distributions of hedge funds across investment styles. The data is from TASS Management Limited. There are 2,016 hedge funds, including 1,407 live funds and 609 dead funds as of July 1999. The reported statistics are for the return history over the three-year period, from August 1996 to July 1999 for live funds, and from October 1995 to September 1998 for dead funds. There are a total of 820 live funds and 64 dead funds that meet this return history requirement.

	LIVE FUNDS						DEAD FUNDS					
STYLE	No.	MEAN	MEDIAN	STD	SKEW	KURT	No.	MEAN	MEDIAN	STD	SKEW	KURT
US Equity Hedge	145	1.66	1.80	5.43	-0.28	3.45	5	0.79	0.91	6.02	-0.29	2.85
European Equity Hedge	23	1.85	1.32	3.77	0.08	3.32	1	1.45	1.08	2.20	0.57	2.29
Asian Equity Hedge	6	0.87	0.57	5.99	0.64	3.55	-	-	-	-	-	-
Global Equity Hedge	29	1.12	1.40	5.11	-0.41	4.85	4	0.44	0.62	2.92	-0.39	3.77
Dedicated Short Seller	7	0.05	-0.48	7.09	0.80	4.01	1	1.24	-1.30	12.74	0.46	3.00
Fixed Income Directional	8	0.82	0.96	2.21	-0.96	6.72	1	0.82	0.95	3.12	0.37	5.44
Convertible Fund (long only)	6	0.95	1.35	3.95	-0.88	4.91	-	-	-	-	-	-
Event Driven	75	1.00	1.13	2.44	-1.04	6.74	5	0.89	1.30	2.58	-2.32	9.86
Non Directional / Relative Value	79	0.84	1.05	1.86	-0.28	5.02	8	0.36	0.46	3.25	-1.40	7.02
Global Macro	15	1.35	1.16	3.87	0.09	3.93	1	1.39	2.47	7.81	-0.57	2.85
Global Opportunity	4	1.25	2.00	9.38	-0.30	3.35	-	-	-	-	-	-
Natural Resources	2	0.17	-0.36	9.80	1.05	6.40	3	0.76	-0.10	7.01	0.90	4.18
Pure Leveraged Currency	22	0.68	0.34	3.21	0.43	4.00	6	0.46	0.72	6.32	0.08	4.39
Pure Managed Future	133	1.31	0.76	5.67	0.29	3.38	10	0.50	0.33	4.44	-0.06	5.03
Pure Emerging Market	72	0.21	1.19	8.06	-0.86	5.22	4	-0.09	0.91	9.46	-2.10	10.37
Pure Property	1	0.31	0.60	3.17	-1.80	8.32	-	-	-	-	-	-
Fund of Funds	193	0.90	0.96	3.21	-0.46	4.30	15	0.12	0.49	3.99	0.03	4.23
Total	820	1.02	1.08	4.27	-0.26	4.12	64	0.56	0.71	4.42	-0.34	4.33

Table 2

Hedge Fund VaR Based on Empirical Return Distribution

This table presents VaR estimates for hedge funds across investment styles. The data is from TASS Management Limited. There are 2,016 hedge funds, including 1,407 live funds and 609 dead funds as of July 1999, out of which 820 live and 40 dead funds meet the three-year return history and equity information requirement. The three-year period is from August 1996 to July 1999 for live funds and from October 1995 to September 1998 for dead funds. Out of the 64 dead funds, only 40 funds have net asset information, which is needed for VaR calculation. The absolute VaR is derived from $VaR = [(R_{mean} - R_{99\%}) \times NAV]$, where VaR is the 99% 1-month VaR, R_{mean} is the mean fund return from historical distribution, $R_{99\%}$ is the cut off return at 99% confidence level from the empirical distribution, and NAV is the total net asset value (equity) of the fund. The relative VaR is the ratio of absolute VaR to fund net assets. The absolute VaR as well as fund assets are in million dollars.

	LIVE FUNDS							DEAD FUNDS						
Style	No.	Asset Size		Abs. VaR		Relative VaR		No.	Asset Size		Abs. VaR		Rel. VaR	
		mean	median	mean	median	mean	median		mean	median	mean	median	mean	median
US Equity Hedge	145	103.3	32.0	13.6	3.9	13.2%	12.1%	2	23.9	23.9	4.8	4.8	19.9%	19.9%
European Equity	23	139.8	81.9	12.1	7.8	8.6%	9.5%	-	-	-	-	-	-	-
Asian Equity	6	131.8	16.4	12.4	2.4	9.4%	14.6%	-	-	-	-	-	-	-
Global Equity	29	362.2	42.5	45.9	6.3	12.7%	14.8%	1	5.9	5.9	0.6	0.6	10.3%	10.3%
Dedicated Short	7	29.9	5.40	2.7	0.6	9.1%	11.8%	1	33.3	33.3	7.9	7.9	23.7%	23.7%
Fixed Income Directional	8	58.1	24.4	3.0	2.8	5.1%	11.5%	1	12.3	12.3	0.9	0.9	7.5%	7.5%
Convertible Fund	6	37.7	31.6	4.0	3.8	10.6%	12.1%	-	-	-	-	-	-	-
Event Driven	75	147.5	79.4	9.8	4.3	6.6%	5.4%	3	167.8	160.0	11.7	9.6	7.0%	6.0%
Nondirect/Relative value	79	197.6	69.5	9.1	2.4	4.6%	3.4%	4	182.0	135.8	16.3	8.7	8.9%	6.4%
Global Macro	15	1031.0	197.2	148.8	11.4	14.4%	5.8%	1	1184.0	1184.0	198.4	198.4	16.8%	16.8%
Global Opportunity	4	159.9	80.8	30.9	13.3	19.3%	16.4%	-	-	-	-	-	-	-
Natural Resources	2	2.5	2.5	0.5	0.5	18.4%	18.4%	3	16.4	3.3	2.0	0.5	12.1%	13.5%
Leveraged Currency	22	69.7	9.8	4.3	0.7	6.2%	6.9%	5	9.8	3.6	1.7	0.8	17.6%	20.6%
Managed Future	133	25.3	5.4	2.6	0.6	10.3%	11.1%	6	16.9	6.3	1.6	0.8	9.5%	12.9%
Emerging Market	72	109.1	23.5	26.1	4.6	23.9%	19.6%	4	27.4	25.4	7.5	5.6	27.3%	21.9%
Pure Property	1	17.1	17.1	1.7	1.7	9.9%	9.9%	-	-	-	-	-	-	-
Fund of Funds	193	75.3	19.7	6.3	1.3	8.3%	6.4%	9	100.3	2.4	4.5	0.2	4.5%	8.6%
Total	820	158.7	26.8	14.2	2.3	8.9%	8.4%	40	9.8	9.0	10.7	1.0	11.0%	11.5%

Table 3
Under-capitalization Based on VaR from Empirical Distribution

This table presents the under capitalization ratios for hedge funds using the empirical return distributions. The data is from TASS Management Limited. There are 2,016 hedge funds, including 1,407 live funds and 609 dead funds as of July 1999, out of which 820 live and 40 dead funds meet the three-year return history and equity information requirement. The three-year period is from August 1996 to July 1999 for live funds and from October 1995 to September 1998 for dead funds. *U-cap*=$(E_{actual}-E_{required})\,/\,E_{required}$ (*U-cap* ratio) represents the degree of under-capitalization, where $E_{required}$ is the required equity that is 3 times the 99% 1-month VaR of the fund (using empirical distribution), and E_{actual} is the actual equity which is taken from the data. A *U-cap* ratio less than zero implies that the actual equity is not sufficient to cover the risk of the portfolio as per the VaR approach.

	LIVE FUNDS						DEAD FUNDS					
STYLE	No. Funds	No. U-Cap	% U-Cap	U-cap Ratio		Avg Lev	No. Funds	No. U-Cap	% U-Cap	U-cap Ratio		Avg Lev
				Mean	Median	Median				Mean	Median	Median
US Equity Hedge	145	2	1.4%	2.6	1.6	1.3	2	0	0	0.7	0.7	1.4
European Equity Hedge	23	0	0	4.7	2.9	1.0	0	-	-	-	-	-
Asian Equity Hedge	6	0	0	2.5	2.3	1.4	0	-	-	-	-	-
Global Equity Hedge	29	1	3.5%	1.9	1.5	1.2	1	0	0	2.2	2.2	1.0
Dedicated Short Seller	7	1	14.3%	2.5	1.8	1.8	1	0	0	0.4	0.4	1.2
Fixed Income Directional	8	0	0	4.8	5.3	1.5	1	0	0	3.4	3.4	-
Convertible Fund	6	0	0	2.1	2.1	1.2	0	-	-	-	-	-
Event Driven	75	0	0	6.2	3.6	1.3	3	0	0	4.5	5.7	-
Non Directional / Relative Value	79	0	0	13.8	6.5	2.6	4	0	0	5.0	5.0	19.2
Global Macro	15	0	0	4.3	3.4	1.6	1	0	0	1.0	1.0	-
Global Opportunity	4	0	0	0.5	0.4	2.3	0	-	-	-	-	-
Natural Resources	2	0	0	1.6	1.6	1.2	3	1	33.3%	1.7	1.7	1.2
Pure Leveraged Currency	22	0	0	5.9	4.8	3.0	5	0	0	3.6	1.2	5.0
Pure Managed Future	133	3	2.3%	2.7	2.1	1.4	6	0	0	2.1	2.0	1.3
Pure Emerging Market	72	16	22.2%	0.8	0.5	1.0	4	2	50.0%	0.9	0.9	1.0
Pure Property	1	0	0	2.3	2.3	1.1	0	-	-	-	-	-
Fund of Funds	193	2	1.0%	4.3	3.4	1.1	9	0	0	4.4	4.0	2.0
Total	820	25	3.1%	4.6	2.5	1.2	40	3	7.5%	3.2	2.2	1.2

Table 4

Under-capitalization Based on Empirical VaR Using Daily Hedge Fund Returns

This table presents the under capitalization ratios for hedge funds using the empirical return distribution. The data is from Hedge Fund Research, Inc. There are 33 hedge funds as of December 2000. Deleting 2 funds with no equity information, we have 31 funds left in our sample. Daily returns range from January 24, 2000 to December 27, 2000. $U\text{-}cap=(E_{actual}-E_{required})/E_{required}$ (*U-cap* ratio) represents the degree of under-capitlization, where $E_{required}$ is the required equity that is 3 times the 99% 1-month VaR of the fund and E_{actual} is the actual equity taken from the data. A *U-cap* ratio less than zero implies that the actual equity is not sufficient to cover the risk of the portfolio as per the VaR approach. The VaR figures are in million dollars, obtained by multiplying the daily VaR by √30.

No.	Style	# of obs.	min	max	mean	std.	skew	kurt	mthly VaR	U-cap	Rel. VaR
1	Equity hedge	216	-5.09	4.41	0.08	0.94	0.3	12.1	10.4	1.37	14.04
2	Equity hedge	233	-4.48	4.82	0.00	1.48	0	3.3	17.9	0.73	19.24
3	Equity hedge	232	-4.71	10.67	0.05	1.64	1.2	10.4	16.6	0.46	22.90
4	Equity hedge	206	-6.49	7.40	-0.16	2.48	0.3	3.4	5.1	0.19	27.91
5	Equity hedge	210	-1.90	2.33	0.03	0.37	1.3	16.9	4.7	5.14	5.43
6	Equity hedge	232	-6.40	4.40	0.08	1.22	-0.6	7.1	9.9	0.75	19.06
7	Equity hedge	145	-7.46	11.54	-0.19	2.74	0.8	5.7	0.4	-0.01	33.69
8	Equity hedge	187	-2.89	4.92	0.27	1.31	0.4	3.5	11.2	1.12	15.76
9	Equity hedge	232	-3.35	4.83	-0.16	1.16	0.5	5.5	4.5	1.07	16.14
10	Equity hedge	215	-6.07	8.42	0.06	1.58	0.6	7.2	11.8	0.73	19.32
11	Convertible arbitrage	234	-1.02	1.73	0.04	0.35	0.2	5.3	0.2	5.51	5.12
12	Convertible arbitrage	235	-3.48	5.29	0.04	0.68	1.7	24.4	9.7	1.83	11.79
13	Convertible arbitrage	231	-1.95	2.28	0.02	0.37	0.5	13.5	10.7	5.34	5.26
14	Convertible arbitrage	233	-1.33	1.74	0.03	0.37	0.3	7.9	2.4	4.83	5.72
15	Convertible arbitrage	236	-2.26	2.08	-0.01	0.45	-0.3	7.4	0.3	3.82	6.92
16	Convertible arbitrage	227	-1.46	0.75	0.02	0.25	-0.9	8.0	2.1	8.43	3.54
17	Convertible arbitrage	234	-2.31	2.30	0.04	0.47	-0.1	10.3	5.1	3.26	7.83
19	Event driven	211	-6.03	5.61	-0.03	1.22	-0.4	7.8	1.9	0.64	20.31
20	Event driven	233	-2.69	2.43	0.05	0.59	-0.2	6.5	4.7	2.56	9.37
21	Event driven	235	-1.76	2.72	0.05	0.53	0.6	7.9	0.1	2.92	8.51
22	Event driven	119	-1.40	1.33	-0.04	0.49	-0.1	3.2	24.3	4.35	6.23
23	Event driven	232	-1.05	1.21	0.05	0.31	0.1	4.5	1.3	6.80	4.28
24	Distressed securities	231	-5.62	7.23	0.02	1.30	0.2	9.4	0.7	0.51	22.00
25	Distressed securities	223	-4.24	2.95	0.00	0.67	-0.7	13.6	3.1	2.84	8.68
26	Distressed securities	223	-4.21	6.31	-0.05	0.87	1.2	17.3	20.0	1.62	12.73
28	Merger arbitrage	119	-2.55	2.77	0.03	0.52	0	14.4	2.9	2.54	9.42
29	Merger arbitrage	236	-1.06	0.82	0.06	0.24	-0.6	6.7	5.6	6.38	4.52
30	Merger arbitrage	235	-1.27	1.44	0.05	0.29	-0.2	8.1	3.3	6.14	4.67
31	Merger arbitrage	235	-0.86	2.14	0.07	0.32	1.1	11.0	1.4	6.89	4.22
32	Merger arbitrage	205	-1.25	1.74	0.05	0.27	1.0	14.4	9.4	8.17	3.63
33	Merger arbitrage	202	-1.25	2.45	0.06	0.36	2.0	18.1	6.9	4.78	5.77
	Median	231	-2.55	2.72	0.04	0.53	0.2	7.9	4.7	2.84	8.68
	Mean	215	-3.16	3.91	0.02	0.83	0.3	9.5	6.7	3.28	11.74

Table 5

Comparison of EVT vs. Empirical VaR using Daily Hedge Fund Returns

This table presents a comparison of the EVT based and empirical VaRs using daily hedge fund returns from from Hedge Fund Research, Inc. There are 33 hedge funds as of December 2000. Deleting 2 funds with no equity information, we have 31 funds left in our sample. The daily returns range from January 24, 2000 to December 27, 2000. The empirical VaR is the 99% 1-month VaR (obtained by multiplying the daily VaR by $\sqrt{30}$) of the fund (using empirical distribution), and the EVT VaR is the VaR that is estimated by using the extreme value theory and 5% of the extreme values on the left tail. The 5% ratio is defined as the ratio of the EVT VaR (estimated using 5% extreme values on the left tail) over the empirical VaR. Similar definitions apply to the 7.5% ratio and the 10% ratio. The VaR figures are in million dollars.

No.	Style	Obs.	Emp. VaR	EVT VaR	5% Ratio	7.5% Ratio	10% ratio
1	Equity hedge	216	10.4	11.9	1.14	1.01	1.02
2	Equity hedge	233	18.0	19.0	1.06	0.91	0.92
3	Equity hedge	232	16.7	15.2	0.91	0.91	0.91
4	Equity hedge	206	5.1	5.3	1.05	1.01	0.96
5	Equity hedge	210	4.7	5.4	1.15	1.04	1.02
6	Equity hedge	232	9.9	10.5	1.06	0.98	1.03
7	Equity hedge	145	0.4	0.4	0.97	1.03	0.99
8	Equity hedge	187	11.2	10.5	0.94	0.96	0.94
9	Equity hedge	232	4.5	4.0	0.88	0.90	0.97
10	Equity hedge	215	11.8	11.9	1.01	1.03	1.04
11	Convertible arbitrage	234	0.2	0.2	0.94	0.97	0.99
12	Convertible arbitrage	235	9.7	9.4	0.97	0.95	0.95
13	Convertible arbitrage	231	10.7	12.4	1.15	1.17	1.21
14	Convertible arbitrage	233	2.4	2.8	1.18	1.09	1.08
15	Convertible arbitrage	236	0.3	0.4	1.17	1.05	1.05
16	Convertible arbitrage	227	2.1	2.4	1.10	1.19	1.20
17	Convertible arbitrage	234	5.0	5.5	1.10	1.00	1.01
19	Event driven	211	1.9	1.9	0.97	0.99	0.99
20	Event driven	233	4.8	4.7	0.99	0.94	0.93
21	Event driven	235	0.1	0.1	0.88	0.96	0.91
22	Event driven	119	24.3	27.2	1.12	1.14	1.08
23	Event driven	232	1.3	1.4	1.08	1.07	1.06
24	Distressed securities	231	0.7	0.7	0.90	0.88	0.88
25	Distressed securities	223	3.1	3.9	1.25	1.21	1.24
26	Distressed securities	223	20.0	19.8	0.99	0.96	0.98
28	Merger arbitrage	119	3.0	3.2	1.11	1.12	1.03
29	Merger arbitrage	236	5.6	6.8	1.21	0.96	0.93
30	Merger arbitrage	235	3.3	3.8	1.14	1.06	1.09
31	Merger arbitrage	235	1.4	1.6	1.14	1.08	1.05
32	Merger arbitrage	205	9.4	10.4	1.10	1.09	1.07
33	Merger arbitrage	202	6.9	8.4	1.22	0.91	0.88
	Mean	215	6.7	7.1	1.06	1.02	1.01
	Median	231	4.8	5.3	1.08	1.01	1.01
	Minimum	119	0.1	0.1	0.88	0.88	0.88
	Maximum	236	24.3	27.2	1.25	1.21	1.24

Table 6
Under-capitalization based on Standard Deviation Based VaR

This table presents the under capitalization ratios for hedge funds using the standard deviation based VaR. The data is from TASS Management Limited. There are 2,016 hedge funds, including 1,407 live funds and 609 dead funds as of July 1999, out of which 820 live and 40 dead funds meet the three-year return history and equity information requirement. The three-year period is from August 1996 to July 1999 for live funds and from October 1995 to September 1998 for dead funds. $U\text{-}cap=(E_{actual}-E_{required})/E_{required}$ (*U-cap* ratio) represents the degree of under capitalization, where $E_{required}$ is the required equity that is 3 times the 99% 1-month VaR of the fund (using standard deviation based VaR), and E_{actual} is the actual equity which is taken from the data. A *U-cap* ratio less than zero implies that the actual equity is not sufficient to cover the risk of the portfolio as per the VaR approach.

	LIVE FUNDS						DEAD FUNDS					
STYLE	No. Funds	No.U-Cap	% U-Cap	U-cap	Ratio	Avg Lev	No. Funds	No.U-Cap	% U-Cap	U-cap	Ratio	Avg Lev
				Mean	Median	Median				Mean	Median	Median
US Equity Hedge	145	2	1.4%	2.3	1.6	1.3	2	0	0	1.1	1.1	1.4
European Equity Hedge	23	0	0	3.8	2.8	1.0	0	-	-	-	-	-
Asian Equity Hedge	6	0	0	1.6	1.4	1.4	0	-	-	-	-	-
Global Equity Hedge	29	1	3.5%	2.0	1.8	1.2	1	0	0	2.7	2.7	1.0
Dedicated Short Seller	7	1	14.3%	1.7	1.0	1.8	1	0	0	0.1	0.1	1.2
Fixed Income Directional	8	0	0	5.5	5.6	1.5	1	0	0	3.6	3.6	-
Convertible fund (long)	6	0	0	2.4	2.7	1.2	0	-	-	-	-	-
Event Driven	75	0	0	7.0	4.9	1.3	3	0	0	6.6	7.9	-
Non Direct/Relative value	79	0	0	11.8	6.7	2.6	4	0	0	7.7	6.3	19.2
Global Macro	15	0	0	3.2	2.7	1.6	1	0	0	0.8	0.8	-
Global Opportunity	4	0	0	0.6	0.5	2.3	0	-	-	-	-	-
Natural Resources	2	0	0	0.8	0.8	1.2	3	1	33.3%	0.7	1.0	1.2
Pure Leveraged Currency	22	0	0	4.5	3.5	3.0	5	0	0	3.5	1.2	5.0
Pure Managed Future	133	5	3.8%	1.9	1.5	1.4	6	0	0	2.3	2.3	1.3
Pure Emerging Market	72	6	8.3%	1.1	0.8	1.0	4	0	0	0.8	0.5	1.0
Pure Property	1	0	0	3.5	3.5	1.1	0	-	-	-	-	-
Fund of Funds	193	0	0	4.5	3.5	1.1	9	0	0	4.0	3.7	2.0
Total	820	15	1.8%	4.3	2.4	1.2	40	1	2.5%	3.3	2.3	1.2

Table 7

Distribution characteristics and VaR of S&P 500 Index

This table presents the Distribution characteristics and VaR numbers for an investment in the S&P 500 index, using S&P 500 returns from January 1970 to July 1999 with 7475 days or 355 months. The 36 months are from August 1996 to July 1999 to match the hedge fund VaR time period. The monthly VaR figures are in million dollars, with an assumed investment of $100 million in the S&P index. The U-cap ratios are computed assuming an equity of $100 million. When using daily returns, the monthly VaR is obtained by multiplying the daily VaR by √30. The EVT VaRs are estimated using 55 of the lowest (extreme) return observations.

Days/months	Mean	Median	Std	Skew	Kurt	VaR (Emp.)	U-cap	VaR (EVT)
7475 (daily)	0.04%	0.04%	0.94%	-1.4	37.4	12.6*	1.80	12.7
355 (monthly)	1.16%	1.31%	4.43%	-0.4	5.3	11.5	1.91	11.6
36 (monthly)	2.31%	4.03%	4.76%	-1.3	5.1	13.6	1.46	-

Figure 1

This figure presents the box and whiskers plots of the ratio of EVT based VaR to the VaR based on the empirical distribution of returns, for the 105 hedge funds with monthly return data for more than 10 years. The three boxes represent three different estimations of the extreme value distribution, using the lowest 5%, 7.5% and 10% of the return observations respectively. The plots show the median ratio, as well as the 5%, 25%, 75% and 95% quantiles of the distribution of the ratio.