Chapter 3
Hedging with Futures Contracts

In this chapter we investigate how futures contracts can be used to reduce the risk associated with a given market commitment. A perfect hedge is a strategy that completely eliminates the risk associated with a future market commitment. To establish a perfect hedge, the trader matches the holding period to the futures expiration date, and the physical characteristics of the commodity to be hedged must exactly match the commodity underlying the futures contract. If either of these features are missing then a perfect hedge is not possible. In such circumstances risk can still be reduced but not eliminated. In this chapter we investigate how risk can be minimized.

In the first section we revisit basis risk, and show how short and long hedges replace price risk with basis risk. Examples of hedges are provided. We then investigate cross hedges with maturity and asset mismatches. Simple hedging strategies that result in minimizing the variance of cash flows when the hedge is lifted are then considered. The final section of this chapter investigates reasons for the firm to hedge. Hedging activities should only be conducted once clear economic reasons for reducing risk have been articulated.

The primary objectives of this chapter are the following:

- To explain how futures contracts can be used to reduce risk;

- To illustrate hedging with detailed examples; and

- To explain why firms may choose to hedge certain types of risk.

Basis Risk Revisited

Recall, that the basis is defined as the difference between the spot and futures price. At date t, we have

$$b(t) = S(t) - F(t) \tag{1}$$

Under the cost-of-carry model, the futures price can be expressed as

$$F(t) = S(t) + C(t, T) - k(t, T) \tag{2}$$

where $C(t, T)$ is the net cost of carry that includes the accrued interest expense, storage and insurance costs less the accrued coupon or dividend yield. $k(t, T)$ is the accrued convenience yield over the period $[t, T]$. Substituting equation (2) into equation (1) we obtain

$$b(t) = k(t, T) - C(t, T) \qquad (3)$$

Equation (3) states that the basis at date t consists of the positive accrued benefits associated with having the inventory on hand, $k(t, T)$, less the accrued cost of carry. As time advances the basis changes. Let $\Delta b(t)$ represent the change in the basis over a small time increment, and let $\Delta k(t, T)$ and $\Delta C(t, T)$ represent the corresponding changes in the convenience yield and cost of carry. Then

$$\Delta b(t) = \Delta k(t, T) - \Delta C(t, T) \qquad (4)$$

For financial assets and investment commodities such as gold, or for consumption commodities that are in ample supply over the period $[t, T]$, the convenience yield is negligible, and the change in the basis is determined by the change in the cost of carry term. This term may change in a predictable way. For example, if interest rates remain constant, the cost of carry term, $C(t, T)$, smoothly converges to zero.

Example

Assume that nationwide inventories of corn are currently large and that the convenience yield is negligible. In this case the futures price is determined by the direct cost of carry. The current spot and futures prices are $S(0) = \$2.06, F(0) = \$2.15$ and $T = 3$ months. The net carry of the futures contract is 9 cents over the 3 month period and the basis is $b(0) = S(0) - F(0) = -\$0.09$. This carry change reflects the interest and storage changes. Assuming this change remains stable over time, then the carry change per month should remain at about 3 cents per month. If this assumption holds, then the basis in one month's time should be -6 cents, in two months time it should be -3 cents, and in the last month it should converge to 0.

The basis for a consumption commodity that is currently in short supply, or anticipated to be in short supply before the delivery date, will reflect a convenience yield. The change in the basis may be less predictable than the corresponding change for a commodity with no convenience yield because of the potential for large unanticipated changes in the convenience yield. In particular, unanticipated imbalances between supply and demand can lead to large shifts in the convenience yield causing the basis to deviate from its predicted level.

When the basis does moves towards zero, it is said to be *narrowing*. Conversely, when the basis moves away from zero it is said to be *widening*. In practice the basis very rarely converges smoothly to zero. However, while the time series behavior of the spot and futures

prices may display significant volatilities, the pricing relationship between the two usually results in the time series of the basis being much more stable. Indeed, the volatility of the basis, will usually be a magnitude smaller than the volatility in the spot or futures price. As we shall see, this low basis variability is very important for establishing hedging strategies.

Short Hedges

Short hedges are usually initiated by traders who own an asset and who are concerned about prices declining before the sales date. To illustrate a short hedge consider a grain elevator operator who is in the business of purchasing and storing grain for future sale. As an example, reconsider the previous corn problem where $S(0) = \$2.06, F(0) = \$2.15$, the delivery date is 3 months away, and the basis is predicted to be fairly stable over the next 3 months, increasing at a rate of 3 cents per month.

If the grain elevator planned on selling its corn in 3 months time, it could eliminate all price uncertainty by selling futures contracts to lock in a specific price. The sale of futures contracts against an inventory of the underlying commodity would then be a *perfect hedge*. However, in this example, we shall assume that the sales date is in 2 months time, a full month earlier than the settlement date. To lock into a sales price for corn, the grain elevator sells a futures contract. After 2 months the grain elevator offsets the transaction in the futures market and sells the corn as planned. The anticipated cash flow at date $t = 2$ is $A(t)$, where

$$
\begin{aligned}
A(t) &= S(t) - [F(t) - F(0)] \\
&= F(0) + [S(t) - F(t)] \\
&= F(0) + b(t)
\end{aligned}
$$

Without the hedge, the anticipated cash flow at date $t, \bar{A}(t)$ say, is given by

$$\bar{A}(t) = S(t)$$

By hedging, the grain elevator has replaced the uncertainty of the commodity price, with the uncertainty of the basis. Since basis risk is smaller than commodity price risk, the grain elevator has reduced risk by this hedging strategy. Indeed, if the cost of carry relationship stays unchanged then after two months the basis should be $-3c$ and, regardless of what the spot price of corn is, the grain elevator anticipates a cash flow of $\$2.15 - \$0.03 = \$2.12$ per bushel of corn.

Example

Reconsider the problem faced by the grain elevator. Consider the net profit that the grain elevator makes from purchasing corn, storing it for two months and hedging this inventory by selling 3 month futures contracts. Let $C^g(0, t)$ be the net accumulated cost incurred by

the grain elevator for financing and carrying the corn inventory over the two month period. The profit at date $t = 2$ months is given by $\pi(t)$, where

$$
\begin{aligned}
\pi(t) &= -[S(0) + C^g(0,t)] + S(t) - [F(t) - F(0)] \\
&= [S(t) - F(t)] - [S(0) - F(0)] - C^g(0,t) \\
&= b(t) - b(0) - C^g(0,t) \\
&= \Delta b(t) - C^g(0,t).
\end{aligned}
$$

Therefore, the net profit is just the change in the basis less the cost of carry for the grain elevator operator. In this example the change in the basis is expected to be 6 cents a bushel. Hence, if the grain elevator's net cost of financing is less than 6 cents a bushel for the two month period, then positive returns can be expected.

Of course the basis may not change in a continuously predictable way determined by the net carry charge. Indeed, due to uncertainty in interest rates or convenience yields, the basis may unexpectedly expand or shrink.

Example

(i) Say interest rates expand unexpectedly. Then the futures price of corn will increase without the spot price changing. In this case the basis may become more negative. This widening basis causes the short position to lose more than anticipated.

(ii) Suppose, due to unanticipated strong demand for corn, the convenience yield increases, driving the futures price down, relative to the spot price. Specifically, the spot price increases by more than the futures price. In this case the basis has become less negative. This narrowing basis causes the short position to profit more than anticipated.

Eurodollar Futures

The Eurodollar (ED) futures contract that trades at the Chicago Mercantile Exchange is based on a 3 month LIBOR rate.[1] These contracts are extremely liquid and the volume of contracts traded makes this market one of the largest. Like stock index futures contracts, the Eurodollar futures contract is settled in cash. The settlement price is based on the 3-month LIBOR rate at the expiration date which is the third Monday of the delivery month. The final settlement price is determined by selecting at random 12 reference banks from a list of 20 major banks in the London Eurodollar market, and identifying their quotes on 3-month Eurodollar time deposits. The two highest and two lowest quotes are dropped,

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[1]LIBOR stands for the London Interbank Offer Rate. It is a widely used benchmark interest rate.

and the arithmetic average computed. Trading in contracts with settlement dates exceeding $2 - 3$ years is quite active.

Let $L(t)$ represent the date t annualized Libor rate in decimal form. The quoted futures price at the settlement date, T, is given by

$$QF(T) = 100(1 - L(T)).$$

Prior to expiration, at date t, the quoted price is

$$QF(t) = 100(1 - IL(t)).$$

Here IL(t) is the *implied* annualized 3-month LIBOR rate. As the settlement date approaches, the implied LIBOR rate converges to the actual spot LIBOR rate.

Example

A June Eurodollar futures contract trades at 95.75. This implies that the implied futures LIBOR rate is $(100 - 95.75) = 4.25\%$. Assume a trader sells this futures contract so as to lock in a rate from June of 4.25%. If the futures price changes to 95.76 then the one basis point increase will be worth $\$1,000,000 \times \frac{1}{100} \times \frac{1}{100} \times \frac{90}{360} = \$25$. The short position will profit if prices decrease, or equivalently as the implied LIBOR increases.

The typical daily gain or loss in one futures contract is in the range $300 - 1000$ dollars. Margin requirements are typically equal to about 4 times the typical daily move.

Example: A Short Hedge with ED Futures

Consider a firm that plans on borrowing one million dollars for three months starting on March 19th which happens to be the last trading day of the March Eurodollar futures contract. The firm has arranged to borrow funds at LIBOR flat. This means that the interest charge will be determined by the 3 month LIBOR index on March 19th. The firm is concerned that interest rates will rise and would like to lock into a fixed rate now. Selling a March Eurodollar futures contract results in replacing the uncertain borrowing rate with a fixed borrowing rate. To see this assume the current 3-month Libor rate is 8%. If Libor remained unchanged, then the interest expense on a one million dollar loan would be $1m \times (0.08) \times (90/360) = \$20,000$. If Libor rates increase one hundred basis points, the expense increases by $100 \times 25 = \$2,500$ to $\$22,500$.

Assume the current implied futures rate for March is $IL(0) = 8.5\%$ and that the firm sells 1 March Eurodollar futures contract. On March 19th the implied LIBOR converges to the LIBOR rate of 9.0%. The profit on the short position is $\pi(T)$ say, where

$$\pi(T) = (IL(T) - IL(0))(90/360)1m$$

$$
\begin{aligned}
&= & (0.090 - 0.085)(250,000) \\
&= & 50 \text{ basis points } \times 25 = \$1,250
\end{aligned}
$$

The actual borrowing cost is $L(T)(90/360)1m = 0.090 \times 250,000 = \$22,500$. The net cost of borrowing is therefore $22,500 - 1,250 = \$21,250$. This corresponds to an effective borrowing rate of 8.5%, which equals the initial implied LIBOR rate.[2] In this example the hedge is perfect and the borrowing cost is exactly 8.5%. If the hedge was lifted prior to date T, then the effective interest rate would be close to 8.5% but would be influenced by basis risk.

Long Hedges

Long hedges are usually initiated by traders who intend purchasing an asset in the future and are concerned that prices may rise in the interim period. To make matters specific, consider a cereal producer who anticipates purchasing corn in $t = 2$ months. Assume the conditions of the previous example. The cereal producer purchases a futures contract so as to lock into a price at date t. At date t, the firm sells the futures contract and purchases the spot commodity. The anticipated cash flow, with this hedge is

$$
\begin{aligned}
A(t) &= & [F(t) - F(0)] - S(t) \\
&= & [F(t) - S(t)] - F(0) \\
&= & -b(t,T) - F(0)
\end{aligned}
$$

Without the hedge the anticipated cash flow at date t is $\bar{A}(t)$ where

$$
\bar{A}(t) = -S(t)
$$

In comparing cash flows note that without the hedge, the firm is exposed to price risk, whereas with the hedge, the risk is reduced to basis risk. Like the previous example, if the firms holding period had coincided with the settlement date, then the basis risk would be eliminated, and the hedge would be perfect.

The above examples indicate that the trader who wants to reduce risk, should choose a futures contract that has a settlement date close to the trading date. In practice, however, holding onto a futures position in the delivery month may be risky. In particular, any long position held in the delivery month, runs the risk of having to take delivery of the commodity, at a location that is inconvenient. Long positions can avoid the delivery risk by canceling their positions before the settlement month begins. Even short hedgers should be careful of holding positions in the delivery month, since the price of the contract can sometimes be erratic. As a result, if possible, hedgers choose a futures contract that has

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[2]In this example, the loan is paid off at the end of the borrowing period (June). The cashflows of the futures contract end in March. Hence, here we are ignoring the interest (expense) on the profits (losses) over the three months between March and June. This approximation is usually small.

a slightly longer maturity than their holding period. For example, if the holding period is November, a December futures contract may be appropriate. Of course, for most contracts liquidity decreases as the settlement date lengthens, and this should also be considered in selecting a contract.

Example

A dental firm has estimated its demand for silver to be 10,000 troy ounces during December and January. The firm is concerned that prices will rise in the interim and would like to lock in today's price of $5.60 without purchasing the silver today. On June 15th, the CBOT's December futures contract trades at $5.90. Since each contract controls 1000 troy ounces, the firm locks into a price of $5.90 by buying 10 contracts. In late November, the spot price has increased to $8.00 and the futures contract is priced at $8.45. At that time the firm sells the futures contract and purchases the silver at spot prices. Exhibit 1 compares this hedging strategy with an unhedged position.

Exhibit 1
Long Hedge in Silver Futures

Unhedged Position	
Cost of Purchasing Silver in November	$80,000

Hedged Position	
Futures	
Buy 10 futures in June at $5.90	$59,000
Sell 10 futures in Nov. at $8.45	$84,500
Profit on Futures	$25,500
Cost of Purchasing Silver in November	$80,000
Net Cost of Purchasing Silver	$54,500

The anticipated net price paid equals the contracted futures price of $5.90 plus the basis in November. The basis was -30 cents and it widened to -45 cents. The resulting cost is $5.90 − $0.45 = 5.45 is lower than one might have predicted because the widening basis created profits for the long position.

Mismatches on Maturities

So far, our problems have involved hedging a risk at an exposure date with a futures contract that has a similar maturity. If there are a sequence of exposure dates, then trading in a sequence of futures contracts may make sense.

Strips of Futures

Assume a firm purchases a commodity at dates $t_1, t_2, ..., t_n$, and is concerned about price increases. This uncertainty can be managed by purchasing a *strip* of forward contracts with maturities coinciding with the exposure dates. By purchasing a strip of forwards, the firm is replacing a stream of uncertain expenses with a stream of fixed costs.

If the exposure extends out over multiple years then the difference between forward and futures contracts becomes important. Recall from the previous chapter that forward contracts can be replicated by a dynamic trading strategy involving futures contracts. The initial exposure from purchasing a forward contract with maturity at date T, is equivalent to the exposure of P(0,T) futures. As a result, hedging a time series of fixed exposures with futures is a dynamic strategy involving the initial purchase of $P(0, t_1)$ futures contracts with maturity t_1, $P(0, t_2)$ futures contracts with maturity t_2, etc. Modifying the simple 1-1 forward hedge ratios to take into account the fact that futures contracts are marked to market is referred to as *tailing the hedge*. Tailing a hedge is an active strategy. In particular, as time evolves the hedge ratios need to be modified.

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Example: Hedging Interest Rate Expenses with a Strip of ED Futures

The treasurer of a firm has arranged to borrow funds in March, June, September and December at rates that are linked to a three month LIBOR. Our particular firm has arranged to borrow at 50 basis points above LIBOR. A firm with a more solid credit ratings might be able to negotiate a cheaper borrowing rate of say 10 basis points above LIBOR. The treasurer expects to borrow 50 million dollars at the beginning of March, June, and September and 60 million dollars next December. It is currently the beginning of December, and the treasurer is concerned that interest rates will rise.

The treasurer decides to sell a strip of futures. In particular he sells 50 March Eurodollar futures, 50 June contracts, 50 September contracts and 60 December contracts. At the beginning of March, the firm would close out its March contract by buying 50 contracts. Similarly, at the beginning of June, September and December, the respective positions would be unwound. Ignoring basis risk this strip of futures replaces the stream of uncertain financing costs with a stream of costs that are based on current implied LIBOR rates.

In this example, the hedge could be improved by tailing it appropriately to reflect the fact that futures contracts are distinct from forward contracts. As an example, take the December leg of the strip. Assume the one year discount rate is 10 percent continuously compounded. Hence the discount factor for one year is $e^{-0.10} = 0.9048$. Rather than sell 60 futures contracts, the firm should sell $0.6048 \times 60 = 54.3$ contracts. Selling 54 contracts now, and increasing this number gradually towards 60, will be more effective since it takes into account the fact that profits and losses accrue to a futures position over the life of the contract.[3]

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[3]When we discussed tailing of hedges we assumed forward prices and futures prices were the same. That is we assumed interest rates were certain. Here we have tailed the hedge in an application involving interest rate uncertainty. As a result, this application is only approximate. We shall investigate hedging interest

Rolling Hedges

Implementing a strip hedge is an effective way to hedge multiple exposures over time. However, it assumes that liquid futures contracts exist with maturities close to the exposure dates. If there are no liquid contracts extending out as far as the exposures, then strip hedges cannot be implemented. When the holding period exceeds the delivery dates of active futures contracts the hedger can initiate a *roll-over strategy*. This involves closing out one futures contract just prior to its delivery month, and then taking the same position in a futures contract with a longer delivery date. The strategy is best illustrated by an example.

Example: **Implementing a Roll-Over.**

In January a firm wishes to establish a short hedge over two years. Futures contracts are only traded with settlement dates every month going out to one year. However, the liquidity of contracts beyond 6 months is questionable. The firm decides to sell 6-month futures and to roll the position over just prior to each delivery month. The sequence of transactions are shown below

Date	Strategy	
January		Sell June Futures
May	Close out June Position	Sell October Futures
September	Close out October Position	Sell February Futures
January	Close out February Position	

The initial spot price was $23 and the 6-month futures contract was $24. The actual futures prices that occurred are shown below

Date	Initial Price	Close Out Price (5-months later)	Profit from Sale of Futures
January	24	22	2
May	21	23	-2
September	23	20	3

The final spot price in January was $19. The commodity dropped $4 over the period. This loss was partially compensated by a net $3 profit on the futures position. Each time the hedge is rolled over, the trading strategy absorbs basis risk. As a result the precision of the hedge deteriorates with the number of roll-overs.

rate claims in more detail in a future chapter.

The next example illustrates some of the difficulties in managing massive long term risks by using successive roll over strategies in shorter term instruments. Roll over strategies can be very useful in reducing risk, but they do not eliminate it, and if the risks are large enough, the firm can still experience cash flow problems.

Example: Maturity Mismatches, and Risks with Roll Overs

In 1993, Metallgesellschaft, a large German engineering and metals conglomerate, revealed that its US trading unit, MG Corp., had incurred losses in energy derivatives of almost one billion dollars. The problem began 18 months prior to the announcement when MG began aggressively marketing gasoline, heating oil and other fuel products on a long term, fixed price basis to its clients. To win business from its competitors, the firm negotiated fixed price contracts for as long as 10 years into the future. Of course, entering into these contracts put MG at high risk. In particular, if oil prices rose, then the firm would have to buy at the higher price and deliver it to its customers at a loss.

To hedge this risk, one of its many strategies was to purchase futures contracts on the NYMEX. Since there was a considerable maturity mismatch, the idea was to roll over the futures contracts into new ones as the old ones expired. MG was confident that these roll overs would not be a problem. However, as the number of fixed price agreements that it entered into with its customers increased, the size of its futures positions grew so large that it exceeded limits on the number of contracts it was allowed to purchase at NYMEX. Moreover, the basis risk at the roll over dates was substantial with reports suggesting that the firm was loosing about 30 million dollars with each successive roll over. At the same time the price of oil began to slip, causing large losses on the futures positions. Because of the timing mismatch between the hedging costs and revenues received from their customers, cash difficulties almost brought the firm to collapse. The problem reached its peak when a margin call of 200 million dollars was issued by NYMEX in late 1993.

Cross Hedging

When *direct* hedges are placed (e.g., a corn position hedged in corn futures) basis risk can be eliminated if the hedge is lifted at the expiration of the futures contract. If no futures contract exist with settlement dates equal to the hedging horizon, then futures with mismatched maturities must be used, and basis risk will be present.

In many cases firms may want to hedge against price movements in a commodity for which there is no futures contract. In this case futures contracts on related commodities whose price movements closely correlate with the price to be hedged can be used. Most hedges that firms may want to establish either have an asset or a maturity mismatch.

Indeed, if there were futures contracts for every asset and date that all traders desired, each market would be extremely illiquid. A hedge that is established with either a mismatched maturity or a mismatched asset or both is referred to as a *crosshedge.* When crosshedging, the trader has to establish the appropriate number of futures contracts to trade, so as to minimize the risk in the hedged position.

Cross Hedging with Maturity Mismatches

So far we have considered hedge positions in which the number of futures contracts were fully determined by the spot position. This hedge is effective if a dollar change in the spot price is exactly offset by a dollar change in the futures price. This assumption is valid when there is no maturity mismatch. However, when there is a maturity mismatch, then the hedging effectiveness can be improved. To see this, assume the hedging period is $[0, t]$, and that the futures contract settles at date T, with $T > $ t. From the cost of carry relationship, we know that

$$F(t) = S(t) + C(t, T) - k(t, T)$$

Here $C(t, T)$ is the accumulated carry change from date t to T which includes the interest expense and storage charges, and $k(t, T)$ is the accumulated convenience yield over the period $[t, T]$. If we assume interest charges are known and that storage costs and convenience yields are proportional to the level of the spot price, and remain constant over time, then the futures price at any date t is related to the spot price by

$$F(t) = S(t)e^{(r-\kappa)(T-t)}$$

where κ is the convenience yield net of direct storage expenses. Notice that the change in futures price for each \$1 change in the spot price, $S(t)$, is $e^{(r-\kappa)(T-t)}$dollars. Since the change in futures price to a one dollar spot price change differs from one, there is no reason a short hedge is best set up by selling a number of futures equal to the spot position.

To make matters specific, assume b futures contracts were sold against the spot commodity at date 0. Then at date t, the anticipated cash flow would be

$$A(t) = S(t) - b[F(t) - F(0)]$$

Substituting the futures price expression into the above equation leads to

$$
\begin{aligned}
A(t) &= S(t) - b[e^{(r-\kappa)(T-t)}S(t) - F(0)] \\
&= S(t)[1 - be^{(r-\kappa)(T-t)}] + bF(0)
\end{aligned}
$$

Now consider a very specific hedge ratio, $b = b^*$, where

$$b^* = e^{-(r-\kappa)(T-t)}$$

Then the coefficient of $S(t)$ is zero and

$$A(t) = b^*F(0)$$

Viewed from time 0, $b^*F(0)$ is certain. Notice that if the holding period coincided with the settlement date $(t = T)$, then there is no maturity mismatch, $b^* = 1$, and all risk is eliminated. At the other extreme, if the hedging period is extremely short, then over the infinitesimal period, the optimal hedge ratio is $b^* = e^{-(r-\kappa)T}$. For intermediate periods, $0 < t < T$, b^* differs between this number and one. The important point here is the fact that the number of futures contracts to sell should differ from that determined by the cash position alone.

Example: Maturity Mismatch in a Silver Hedge

Suppose a photographic paper manufacturer must purchase silver at the end of January. It is currently June. The firm wants to hedge against increases in silver by going long futures. Assume the nearest futures contract that expires beyond January is the March contract. Thus the firm must use a crosshedge that has a two month mismatch. The current March futures price is \$5.90. The interest rate is 12% per year, and storage costs are negligible. The appropriate hedge ratio is b^* where $b^* = e^{-(r-\kappa)(T-t)}$, where κ is the convenience yield in January. The firm believes that while silver futures should normally be close to full carry, a higher convenience yield may materialize in January. This prediction is further confirmed when the implied convenience yield for silver over the January to March period, extracted from the term structure of futures prices, yields an estimate of $\kappa = 3\%$ per year. Under the scenario that the convenience yield, κ, is 3% in January, $b^* = e^{-(r-\kappa)(T-t)} = 0.985$ Further, since each futures contract on the CBOT covers 1000 troy ounces, and since 50,000 troy ounces are required, the firm requires about 49 contracts.

If the convenience yield is 3% when the hedge is lifted, then the hedge will almost be perfect. The firm can gauge the risk of this long hedge by specifying different realized convenience yields in January and investigating the resulting hedging risk. For example, Exhibit 2 shows the effective hedging costs for a variety of spot prices in January, under the assumption that in January convenience yields have increased to 10%. The analysis indicates that if the convenience yield increases by 2% above what had been anticipated, the hedge remains very effective.

Example

A firm holds a large inventory of silver. It is known that an important announcement is going to be made in the next 48 hours, and it could have an adverse affect on silver prices. To hedge price risk over this short time period, the firm plans to sell futures contracts. The appropriate hedge ratio is given by $b^* = e^{-(r+u-\kappa)T}$. With interest rates at 12% per year, a convenience yield that has been robust at 3%, and negligible storage charges, the hedge ratio, using a contract with 3 months to settlement, is $b^* = e^{-(0.12-0.03)0.25} = 0.977$.

Exhibit 2
Effective Purchase Costs under A Long Hedge

Spot Price	5.60	5.70	5.80	5.90	6.0	6.10	6.20
Projected Futures Price	5.618	5.719	5.819	5.919	6.020	6.120	6.221
Profit from Futures	-0.281	-0.181	-0.081	0.019	0.1200	0.220	0.321
Profit from 49 Futures (in thousands)	-13.784	-8.867	-3.95	0.965	5.882	10.798	15.714
Cost of Spot (in thousands)	280	285	290	295	300	305	310
Effective Purchase Price (dollars per ounce)	5.875	5.877	5.879	5.881	5.882	5.884	5.886

Risk Minimizing Hedge Positions

The above example illustrates that, for a mismatched maturity, a hedge ratio of $b = 1$ may not reduce risk to a minimum. In our example the sale of $b^* = e^{-(r+u-\kappa)(T-t)}$ contracts reduced risk further. This raises the question as to what the best hedge ratio is. To answer this question, we first have to have a very precise measure of risk so that different strategies can be compared. In this section we assume the goal is to reduce risk as much as possible, and that the measure of risk is given by the variance of anticipated cash flows when the hedge is lifted.

Once again, standing at date 0, consider a hedge involving the sale of b futures against the spot commodity. The anticipated cash flow at date t is

$$
\begin{aligned}
A(t) &= S(t) - b[F(t) - F(0)] \\
&= S(0) + S(t) - S(0) - b[F(t) - F(0)] \\
&= S(0) + \Delta S(t) - b\Delta F(t)
\end{aligned}
$$

The variance of cash flows is given by[4]

$$
\begin{aligned}
Var_0[A(t)] &= Var_0[S(0) + \Delta S(t) - b\Delta F(t)] \\
&= Var_0[\Delta S(t) - b\Delta F(t)] \\
&= Var_0(\Delta S(t) + b^2 Var_0 \Delta F(t) - 2b Cov_0(\Delta S(t), \Delta F(t)
\end{aligned}
$$

The idea is to choose the number of futures contracts to sell, b, such that the risk, as measured by variance, is reduced to its minimum possible level. Exhibit 3 shows the variance
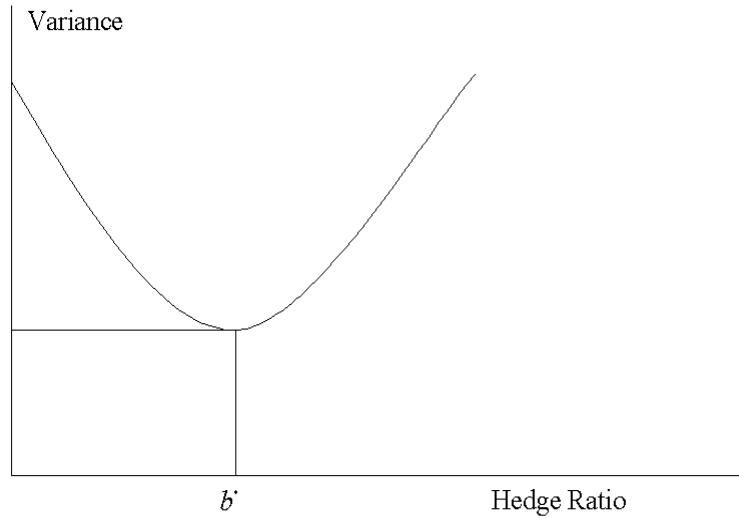
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[4]Recall the variance equation, $\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(X,Y)$. Here, $a = 1$, $X = S(t)$, and $Y = F(t)$.

of cash flows for different b values, under the assumption that the variance and covariance terms are given.

To obtain the minimum risk position, it can be shown that b must be chosen as follows.

$$b = b^* = \frac{\text{Cov}_0[\Delta F(t), \Delta S(t)]}{\text{Var}_0(\Delta F(t))} \qquad (5)$$

Exhibit 3
Risk Minimizing Hedge Positions

The optimal hedge ratio depends on the covariance between the changes in cash and futures prices, relative to the variance of futures price changes.

The correlation coefficient between the changes in the futures and spot prices, ρ, is defined as

$$\rho = \frac{Cov(\Delta F(t), \Delta S(t))}{\sqrt{Var(\Delta F(t))Var(\Delta S(t))}}$$

Hence equation (5) can be rewritten as

$$b^* = \rho\sqrt{\frac{Var(\Delta F(t))}{Var(\Delta S(t))}} = \rho\frac{\sigma_F}{\sigma_S}$$

where $\sigma_S = \sqrt{(Var(\Delta S(t))}$ and $\sigma_F = \sqrt{(Var(\Delta F(t))}$ are the standard deviations of the changes in prices over the holding period $[0, t]$.

Example

Suppose a trader has a holding period of 2 months. Assume the standard deviation of spot prices over two month periods is $\sigma_S = \$0.18$ and the volatility of the futures contracts over the same period is $\sigma_F = \$0.29$. The correlation of the two changes in prices is $\rho = 0.85$. The optimal hedge ratio is then given by:

$$b^* = 0.85 \times \frac{0.29}{0.18} = 1.369. \tag{6}$$

This means that the size of the futures position should be 1.369 times the size of the traders exposure in a two month hedge.

Estimating the Minimum Risk Hedge Position

The parameters for the minimum hedge are usually estimated using historical data. If the hedge is to be in place over a period of time $[0, t]$ say, then historical data on the spot and futures price must be collected over non overlapping periods of width t. For example, if the hedge is to last 2 months, then a time series of spot and futures prices with two month increments should be used. In practice, this limits the number of data points that are available for the analysis and data is collected over shorter time horizons.

Let $\{s_1, s_2, s_3, \ldots\}$ and $\{f_1, f_2, f_3, \ldots\}$ represent the closing daily prices of the spot and futures prices. The time series of futures prices that is used in the analysis should corresponds to the maturity contract that will be used in the hedge. Using these time series, we would like to estimate the terms in equation (5). Let $\Delta f_k = F_k - F_{k-1}$ and $\Delta s_k = S_k - S_{k-1}$ denote the price increments on the kth day. Viewed from date 0, the price change over t days, $\Delta F(t)$ and $\Delta S(t)$ can be expressed as the sum of t daily changes. Then we have:

$$
\begin{aligned}
Var_0\{\Delta F(t)\} &= Var_0\{F_t - F_0\} = Var_0\{\Delta f_1 + \Delta f_2 + \cdots + \Delta f_t\} \\
Var_0\{\Delta S(t)\} &= Var_0\{S_t - S_0\} = Var_0\{\Delta s_1 + \Delta s_2 + \cdots + \Delta s_t\} \\
Cov_0\{\Delta F(t), \Delta S(t)\} &= Cov_0\{F_t - F_0, S_t - S_0\} \\
&= Cov_0\{\Delta f_1 + \Delta f_2 + \cdots + \Delta f_t, \Delta s_1 + \Delta s_2 + \cdots + \Delta s_t\}
\end{aligned}
$$

Then equation (5) can be rewritten as

$$b^* = \frac{Cov_0\{\Delta f_1 + \Delta f_2 + \cdots + \Delta f_t, \Delta s_1 + \Delta s_2 + \cdots + \Delta s_t\}}{Var_0\{\Delta f_1 + \Delta f_2 + \cdots + \Delta f_t\}} \tag{7}$$

In order to estimate this equation we have to make some assumptions on the evolution of the series of price increments. If each of the price increment series are uncorrelated and have the same variances, then

$$
\begin{aligned}
Var_0\{\Delta f_1 + \Delta f_2 + \cdots + \Delta f_t\} &= t Var_0\{\Delta f\} \\
Var_0\{\Delta s_1 + \Delta s_2 + \cdots + \Delta s_t\} &= t Var_0\{\Delta s\} \\
Cov_0\{\Delta f_1 + \Delta f_2 + \cdots + \Delta f_t, \Delta s_1 + \Delta s_2 + \cdots + \Delta s_t\} &= t Cov_0\{\Delta f, \Delta s\}
\end{aligned}
$$

where Δf and Δs are random variables representing the price change in any day. Substituting these expressions into the above equation, leads to

$$b^* = \frac{\text{Cov}_0\{\Delta f, \Delta s\}}{\text{Var}_0\{\Delta f\}} \tag{8}$$

The time series of data can then be used to estimate the numerator and the denominator.

Example

A farmer wants to hedge against falling prices. The cash crop will be brought to market in one months time. The following information is collected on weekly price changes.

Exhibit 4: Estimating a Hedge Ratio

Week		Spot Price Change	Futures Price Change
1		-2.05	-0.8
2		7.2	10.1
3		2.5	-2
4		-1.7	-0.5
5		2.1	1.8
6		-8.35	-8.2
7		0.75	1
8		-3.25	-4.1
9		-2.15	-3.9
10		-2.7	-0.6
11		1.7	-0.1
12		-1.75	-1.3

	sdev S		3.854	sdev F	4.331
		Cov(S,F)	14.830		
		Cor(S,F)	0.969		
		b	0.791		

Regression Statistics

R Square	0.790
n	12

ANOVA

	df	SS	MS	F	p value
Reg.	1	128.990	128.990	37.526	0.000
Residual	10	34.374	3.437		
Total	11	163.364			

	Coef	Std. Error	t Stat	P-value
Intercept	-0.075	0.543	-0.138	0.893
X Variable	0.791	0.129	6.126	0.000

The above table shows that the hedge ratio is $b^* = 0.791$.

Regression Models for Minimum Variance Hedging

Actually, the estimate of the hedge ratio, b^*, can also be obtained by regressing the daily spot price changes against the daily futures price changes, and identifying the least square estimate of the slope. Specifically, consider the model obtained by regressing daily changes in futures prices against daily changes in spot prices.

$$\Delta S(t) = \alpha + \beta \Delta F(t) + \epsilon(t)$$

If the error terms, $\{\epsilon(t), t = 1, 2, \ldots\}$, have zero means, the same variances, and are uncorrelated, then the estimated slope of this regression equation, is the appropriate estimator for the hedge ratio.

Example

The above exhibit shows the output from a regression analysis. Notice that the slope of the regression equation is exactly the same value as the hedge ratio computed earlier.

Hedging Effectiveness

In order to measure the effectiveness of the hedge, it is first necessary to establish the risk of an unhedged position. This is captured by the variance of the price of the commodity underlying the futures contract. Let σ_s^2 represent the variance of the price changes. Now, consider the variability of the price changes of the optimally hedged position. While the futures contracts explain some of the variability, some randomness still exists. This variability is accounted for by the error term in the regression equation, is called the basis error, and is denoted by σ_ϵ^2. The ratio $\sigma_\epsilon^2/\sigma_s^2$ can range from 0 to 1. If it were 0, then there is no basis error, and a perfect hedge can be constructed. If the ratio were 1, then none of the risk can be hedged away. The effectiveness of the hedge is captured by ρ^2 where

$$\rho^2 = 1 - \sigma_\epsilon^2/\sigma_s^2$$

In practice historical data is used to estimate $\sigma_\epsilon^2, \sigma_s^2$ and ρ^2. It turns out that in the regression analysis of spot against futures price changes, the R squared measure is the estimate of ρ^2. As an example, from the R^2 information in the last exhibit, we see that the regression accounted for about 79% of the variance of the unhedged position.

Holding the length of the estimation period constant, it would appear that using more frequent data, such as daily data, would provide better estimates than using less frequent data, such as weekly data. Indeed, one could use daily variance and covariance changes to obtain weekly or monthly estimates. Unfortunately, from a practical perspective, there

are some problems when using data that is collected very frequently. First, transaction prices usually occur at the bid or the ask price. As the time increments become shorter, the contribution to price variability made by the random movement between bid and ask prices increases, and this distorts the true measure of price variability. Second, if short periods are used, the problem of making sure that the futures and spot prices are current increases. In particular, if the futures and spot commodity are not traded at the same frequency, then the two prices may not reflect the same information. The bid ask spread issue together with lack of simultaneous market prices can lead to the error terms in the regression equation displaying serial correlation which increases as the time increments get smaller. Finally, in some markets there are effects due to the day of the week. For example, the change of prices over a weekend may be different from the change in prices over any day. Also Monday's return may be quite different from other business days. Seasonality factors can cause the variances of daily price changes to vary. As a result of these factors, using daily data may not be advantageous over using weekly data.

Ex Ante Hedge Ratios versus Ex Post Hedging Results

In the above example, the hedge ratio was constructed using historical data, and then the effectiveness of the hedge was established using the very same data. In practice, the hedge ratio will be computed using historical data, and then applied to a current situation. Hopefully, the relationships in the past will remain fairly stable and the hedge will be effective. However, the hedge is not likely to be as effective as measured by R^2, because some unanticipated changes in the structure are likely to occur. Indeed, evaluating the hedge in-sample as we have done is quite likely to overstate the actual hedging effectiveness because effectiveness is measured on the same set of data from which the regression equation was derived!

In practice of course, one would estimate the hedge ratio using the most recent data, and then implement the hedge. The effectiveness of the hedge is then determined as uncertainty reveals itself.

To validate the real usefulness of the hedge we therefore should examine how well the model performs when given new data. We could split our data set into two parts, using the first part to establish the hedge ratio, and then assessing the performance of this hedge ratio on the second data set.

In many cases the hedging effectiveness out-of-sample is significantly different from the in-sample measure. If this is the case, and if the hedge ratio computed using the second data set alone is quite different than that obtained from the first data set, then the hedge ratio is unstable and one must proceed with caution. Oftentimes the assumptions of the regression model are being violated, or the estimates are highly sensitive to just a few data points. Researchers have shown, however, that this method works quite well for many consumption commodities, but the level of autocorrelation should be closely monitored. Specifically, if the residuals in the regression analysis display certain time varying patterns then more sophisticated estimation procedures must be used. One statistic that can be used to test for autocorrelation is the Durban Watson statistic which is usually reported as part of the

regression output.

Hedging Stock Portfolios

The above procedure of regressing changes in spot prices against changes in futures prices yields reasonable estimates for a hedge ratio if the assumption that the relationship between the price changes of spot and futures remains somewhat stable over time. While this assumption seems to be well satisfied by many consumption commodities, it has been found to be lacking for financial assets. For example, the price changes of stocks are often serially correlated, and their variances are not constant, but rather fluctuate according to their level of prices. For such securities, the relationship between the *rates* of change of prices and *rates* of change in futures may be more stable. In such cases, it might be preferable to run a rate-of-change regression:

$$\frac{\Delta s_t}{s_t} = \alpha + \beta \frac{\Delta f_t}{f_t} + \epsilon_t$$

Recall that the hedge ratio is trying to capture the sensitivity of the changes in the spot price to changes in the futures price when the hedge is lifted. In the above equation, the slope β is capturing the percentage change in the spot relative to a percentage change in the futures price. To back out the appropriate hedge ratio, then, requires multiplying the estimate of β by the ratio of the current spot price relative to the current futures price. That is:

$$\text{Hedge Ratio} = b^* = \beta \frac{S(0)}{F(0)} \tag{9}$$

Notice, that the hedge ratio varies according to the spot and futures price. As a result, over the holding period the hedge may need to be adjusted as the spot to futures ratio changes.

Hedging Applications in Stock Markets

Exhibit 5 shows price information on stock index futures as reported in the Wall Street Journal. Recall that a stock index futures contract is a legal commitment to deliver or receive the dollar value of the index, multiplied by a given multiplier, at a predetermined future date at a predetermined cost. Of the stock index contracts shown, the $S\&P500$ futures contract is the most active.

Example

Consider the situation of an owner of a well-diversified portfolio who anticipates a short-term decline in the stock market. Liquidation of the portfolio for the short term is not realistic because of high transaction costs, dividend income, and tax consequences. Without financial hedging devices, the owner may have to bear the risk of a short-term declining market. With stock index derivatives available, the investor may hedge against market-related risk by selling stock index futures. The effectiveness of the hedge depends on the

Exhibit 5
Stock Index Futures Price Information from the Wall Street Journal

degree of correlation between the index and the portfolio. In a worst case situation, the portfolio value could depreciate while the market index appreciates. However, this event is unlikely if the stock index is highly correlated with the portfolio.

To make matters specific, assume the portfolio manager owns three stocks.

Stock	Shares Owned	Stock Price	Beta
A	1m	40	1.2
B	2m	20	1.3
C	2m	10	1.1

The value of the investments are shown below

Stock	Value of Investment	Fraction of Wealth
A	40m	0.4
B	40m	0.4
C	20m	0.2

The beta value of the portfolio is $\beta_p = 0.4(1.2) + 0.4(1.3) + 0.2(1.1) = 1.22$

Assume the manager is concerned about future market developments and wishes to reduce the risk associated with this portfolio. The manager does not want to liquidate the portfolio and purchase government bonds because he believes market uncertainties will clarify over the next few months. Rather, he decides to reduce the beta value using stock index futures.

Recall that the beta value of the portfolio captures the sensitivity of expected returns to the underlying index. Assume for the moment that the index used to compute the beta values was the $S\&P500$ index. The analysis indicates that for every 1% change in the $S\&P500$ index, the portfolio is expected to appreciate by 1.22%. Assume the $S\&P500$ index is at 600. Given a multiplier for the futures contract is 250, each contract controls $600 \times 250 = \$150,000$. With no maturity mismatch, and a beta value of 1 the minimum variance hedge is therefore to sell $100m/150,000 = 666.66$ futures. Since the beta value is 1.22, additional futures should be sold. In particular, a total of $666.6 \times 1.22 = 813.3$ futures need to be sold.

If the beta values of the stocks were based off an index other than the $S\&P500$ index then additional adjustments have to be made. For example, assume the stock betas were computed against some index, I say. A regression analysis relating the weekly percentage changes in the index I to the weekly percentage changes in the $S\&P500$ index yielded a slope of $\beta_{I,S\&P} = 0.8$. This means that for every 1% change in the $S\&P$ index, the index is expected to change by 0,8%. The modified hedge ratio of 813.3 now has to be adjusted to $813.3 \times 0.8 = 651$ contracts. In general then, with no maturity mismatch, the hedge ratio is given by $\beta_{pI} \times \beta_{I,S\&P}$ where β_{pI} is the beta value of the stock portfolio with respect an index I.

A primary advantage of selling futures to reduce the beta value over a period of time, rather than selling stocks and buying bonds, is the ease in which the former transaction can be done. In particular, the transaction costs involved in buying and later selling 1 futures contract may be of the order of \$14. This implies a charge of \$9,114 for the above transaction. In contrast, the round trip cost of selling \$100m worth of stock and purchasing it back at a future date, may cost 0.1% of the value of the portfolio, or \$100,000. Unless the futures contracts are significantly mispriced, the commission charges certainly favor the futures strategy.

Example: Intermarket Spreading with Futures

The broad-based market indices are highly correlated with each other. However, if the returns on one index are regressed against the returns on another, the resulting slope could be significantly different from 1. For example, if the $S\&P500$ is used as the base index and the New York Stock Exchange Index is regressed against it, a beta estimate of, say, 1.23 could be obtained. This implies that the New York Stock Exchange could rise approximately 23 percent more than the $S\&P500$ Index in a bull market. An investor who perceives a bullish market could buy the "high beta" futures contract and sell the "low beta" futures in anticipation of the spread widening in favor of the high beta index. Conversely, in a declining market, the low beta futures could be bought and the high beta futures sold in anticipation of the spread narrowing.

Cross Hedging With Asset Mismatches

Let $P(t)$ be the spot price of commodity P at time t. No futures contracts exist for this commodity. Let $S(t)$ be the spot price of commodity S. Futures contracts trade on this commodity and the price movements of P are highly correlated to those of S. Consider a trader who holds an inventory of P and is concerned that prices will fall. To hedge this risk the trader sells b futures on S. The anticipated cash flow at the sales date t is $A(t)$ where

$$
\begin{aligned}
A(t) &= P(t) - b[F(t) - F(0)] \\
&= P(0) + [P(t) - P(0)] - b[F(t) - F(0)] \\
&= P(0) + \Delta P(t) - b\Delta F(t)
\end{aligned}
$$

Viewed from time 0, the first two terms are uncertain. Hence

$$
\begin{aligned}
Var_0[A(t)] &= Var_0[\Delta P(t) - bF(t)] \\
&= Var_0[\Delta P(t)] + b^2 Var_0[\Delta F(t)] - 2bCov_0[\Delta P(t), \Delta F(t)]
\end{aligned}
$$

This variance reaches a minimum when $b = b^*$ where

$$
b^* = \frac{\text{Cov}_0[\Delta F(t), \Delta P(t)]}{\text{Var}_0[\Delta F(t)]}
$$

Now first consider the case where there is no maturity mismatch. In this case $F(T) = S(T)$ and

$$
b^* = \frac{\text{Cov}_0[\Delta S(T), \Delta P(T)]}{\text{Var}_0[\Delta S(T)]}
$$

The easiest way to estimate $b*$ is to estimate the slope of the following regression equation

$$
\Delta P(T) = \alpha + \beta \Delta S(T) + \epsilon(T)
$$

If the error terms are uncorrelated and have mean 0, then the estimate of the slope is the estimate of b^*. Notice, that since their is no maturity mismatch the relationship of interest is one between the price changes of the two assets. As a result, the regression analysis does not require futures data.

If there is a maturity mismatch as well, then, to estimate the value of b^*, a regression analysis could be done where $\Delta P(t)$ is the dependent variable and $\Delta F(t)$ the independent variable. That is

$$
\Delta P(t) = \alpha + \beta \Delta F(t) + \epsilon(t)
$$

The estimate of the slope is the estimate of b^*.

As a matter of fact, most hedges are cross hedges. A trader hedging gold using gold futures has a cross hedge if the underlying gold that is held is not the deliverable for the COMEX futures contract. In many cases no effective cross hedges exist. For example, a mango farmer will not be able to find a traded futures contract that provides an effective hedge. Even a commodity such as barley has weekly price changes that are not very well correlated with wheat, soybeans or corn. Indeed, a multiple regression

analysis of the changes in barley prices against changes in the prices of many of the grains fails to produce highly significant predictors. US barley users will thus find it difficult to hedge price risk of barley. Although barley futures are traded at the Winnipeg futures exchange in Canada, such contracts are not that liquid, and using them introduces foreign exchange risk into the analysis. As a result opportunities still exist for exchanges to introduce new contracts that are useful in that they expand the set of securities that permit price risks to be better managed. Finally, cross hedges can be constructed in which more than one futures contract is used. For example, a portfolio of corporate bonds could be hedged using futures on Treasury securities and futures on stock indices.

Other Approaches to Establishing Hedge Ratios

There are other analytical, as opposed to statistical, methods for setting up hedge ratios that minimize risk. For example, when the underlying commodity underlying the futures contract is an interest sensitive asset then specialized procedures exist. We shall defer discussion of these procedures until future chapters. In addition, we have only investigated static hedging schemes. These are schemes where the hedge is set up at date 0, and then not revised over time in response to the release of new information. Dynamic hedging strategies will be described in future chapters.

The Rationale for Hedging By Corporations

Most firms have no particular expertise in predicting interest rates, exchange rate movements, commodity prices etc. At first glance it appears quite obvious that such firms should hedge these risks so as to be able to focus on their main activities. By hedging unwanted risks they avoid surprises. While such rationale may be true in many cases, a more careful case for hedging should be made on grounds other than risk aversion.

The overall objective of the firm is to maximize its value. At a conceptual level, the value of the firm, V_0, is given by the present value of all future expected cash flows. That is,

$$V_0 = \sum_{i=0}^{n} \frac{E\{CF_i\}}{(1+\rho)^i}$$

where $E\{CF_i\}$ is the expected net cash flow in period i and ρ is the appropriate discount rate for the cash flow. The use of derivative products to manage financial risk is justified if the value can be increased by either increasing expected net cash flows or decreasing the discount rate.

Since individuals are risk averse, at first glance one might suspect that they would want managers of the firm to reduce financial price risks by hedging. However, this is not the case. For individual shareholders, risks such as interest rate risk, commodity price risk and foreign exchange risks are diversifiable. That is, these risks can be eliminated by holding well diversified portfolios. Therefore, hedging by itself will not increase the value of the firm by reducing the discount rate for cash flows. Risk aversion can only be used as a rationale for hedging if the owners of the firm do not hold diversified portfolios. This may well be the case for closely held corporations. In the context of equation (21), for hedging to be beneficial to shareholders of a widely held firm, it must be the case that it somehow increases the expected net cash flows.

Of course, hedging is simply one of the firm's financial policies. The question then, is how can any financial policy impact the real cash flows of the firm. In a famous proposition, Miller and Modigliani showed that in a world with no transaction costs and taxes, a firm with a given investment policy could not increase its value by changing its financial policy. That is under their assumptions, financial policies are irrelevant. The proposition is built on the premise that anything a firm can do in financial markets, its shareholders can do on their own accounts. So if it is advantageous for the firm to hedge using futures contracts, then individual shareholders could just as easily hedge. This being the case, there is no reason for investors to pay premiums for shares to be hedged, when they can do it at no cost.

The Miller-Modigliani proposition implies that if hedging activities are to be relevant, in the sense that they have an impact on the value of the firm, then it must be the case that financial policies impact transaction costs, taxes, or the investment decisions of the firm. In addition, the proposition assumes that individual shareholders have complete

information about the firm, and hence are able to make decisions about whether to hedge risks as they materialize. We now look at how these features lead to motives for hedging.

Hedging and Taxes

The tax schedule is a convex function, illustrated in exhibit 6. Consider a firm that has a certain pretax income of $\$x$. The taxable income on that is shown in the exhibit as t_X^0. Now consider a second firm which has a probability of 0.5 of generating pretax income of $x - y$, and a probability of 0.5 of having pretax income of $x+$y. The tax is either t_{X-Y} or t_{X+Y}, and the expected tax is therefore $(t_{X-Y} + t_{X+Y})/2 = t_X^Y$ say.

<div align="center">

Exhibit 6:
The Convex Nature of the Tax Schedule
</div>
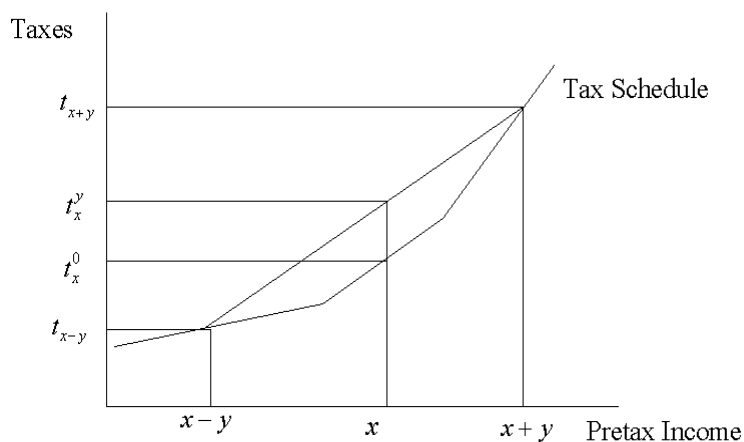
Note that because of the convexity of the tax code,$t_X^Y > t_X^0$. The greater the convexity, and the greater the uncertainty of the income, here captured by y, the greater the difference in expected taxes. The example shows that firms may want to reduce the uncertainty of their revenues by hedging so as to reduce expected taxes.

Transaction Costs and Financial Distress

By hedging, the firm can reduce the likelihood of outcomes that head the firm into financially distressed states. The costs of distress include the direct legal, accounting and

reorganization fees as well as indirect costs associated with higher contracting costs with customers, employees and suppliers and lost business. If the unhedged firm has a high probability of entering financially distressed states, and if the costs of being in financial distress are very high, then the benefits of hedging will be high.

Transaction Costs and Contract Sizes

Futures contracts are quite large and often sized to meet the needs of firms rather than individual investors. Firms may be able to transact at wholesale prices. That is, firms may be able to reduce transaction costs and commissions by establishing relationships with brokerage firms or even by setting up their own trading firm. As a result, the firm may be better equipped to manage the hedging activities.

Asymmetry of Information

Of course firms may have more information than shareholders concerning specific risks. For example, in order to hedge commodity risks, the individual shareholders need to know the timing and sizes of the commitments. in some cases, for strategic reasons, the firm may not want to publicize these commitments for their competitors to learn. As a result, the firm is in a better position to hedge than individual shareholders.

Conflicts of Interest between Managers and Owners

Managers of a firm may choose to hedge and reduce risk because they are looking after their own interests, not necessarily those of the owners. In particular, managers may be adverse to risk since bad outcomes could mean loss of their jobs. Hence managers may be more likely to hedge, even if it is not in the best interests of the owners. Usually the owners are the shareholders who are unable to monitor all the actions of the managers and give them some authority to take actions on their behalf.

In summary, we have provided several reasons for why firms may choose to hedge. In the design of any particular hedge for a firm, it is important to evaluate why the firm wants to accomplish by hedging.

Hedging and Competitors

In some circumstances, the use of futures contracts can actually lead to the creation of more risk, not less. As an example, consider a fairly competitive industry in which prices of raw materials fluctuate up and down but are typically passed on to the consumers in the form of higher or lower prices. In such an industry, the profit margin remains stable despite large price fluctuations.

Assume that in this market a particular firm decides to hedge the prices of its raw materials by purchasing futures. If prices rise, then the price of outputs tend to rise as the firms competitors pass on the increased costs. In this case the long hedger obtains larger profits. If however, prices of raw materials decrease, the hedger looses on the position. Moreover, since the finished goods prices are lower, relative to the competition, profits are

lower.

In this example, the firms in the industry had a built in hedge provided by the fact that all changes in costs could be passed onto the consumers. By hedging the inputs the firm destroys this natural hedge and actually ends up with more volatile net cash flows. Of course, if the output prices are fixed, as is the case in many long term supply arrangements, or if all the changes in costs are not passed onto the consumers, then no natural hedge exists, and appropriate hedging strategies can be designed to reduce risk.

Conclusion

This chapter has been concerned with the design of short and long hedges. In particular we investigated perfect hedges and cross-hedges that had maturity and/or product mismatches. We also investigated how minimum variance hedges could be constructed. The methodology used to establish the optimal hedge ratio in this chapter is a statistical approach and is somewhat generic in that it can be applied to many consumption commodities as well as a few financial assets. However, in some cases, such as hedging bond or stock portfolios, specialized methods are available. As an example, consider a portfolio manager who wishes to use futures to hedge market related risk in the underlying portfolio. Given the beta value of the portfolio simple hedging strategies can be devised using alternative procedures. Some of these procedures will be discussed in future chapters. Finally, we provided a brief discussion of why firms hedge.

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For additional examples of hedging applications refer to the publications of the CBOT and other exchanges. Also, there are many examples discussed in *Risk* magazine. The article by Nance, Smith and Smithson provides some empirical tests of factors that effect the firm's decision to hedge.

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Exercises

(1)   A photographic paper manufacturer has estimated that the firm will require 50,000 troy ounces of silver during December and January. The firm is concerned that prices of silver will rise, and would like to hedge against that risk. The current date is July 1*st*. The CBOT's December silver futures contract is trading at $5.80 per troy ounce. Each contract controls 1000 troy ounces.

a)   Establish the position the manufacturer should take.

b)   Assume in the middle of November, silver is selling at $7.80 per troy ounce, and the December futures contract is at $8.10. At this time the firm purchases silver in the spot market. Compare the net cost of purchasing the silver for the hedged and unhedged position.

(2)   The CME is the world's largest futures trading center for non storable commodities, one of which is live cattle futures. In November, a cattle producer buys feeder cattle with the intent to feed them for future sale in April. To cover all production costs and guarantee a profit, the producer will need to sell the cattle at $65/cwt. The current April live cattle futures price is $70/cwt. and the basis is $-\$3.00$.

a)   Set up a short hedge position for this cattle producer and analyze it assuming that the futures price at the beginning of April, when the contract is bought back is at $65 and the basis has narrowed by $1.0.

b)   Repeat (a) and compute the realized price in April if the futures price in April is $72 and the basis has remained unchanged.

(3)   A farmer who has planted soybeans for November harvest estimates that to profit he has to sell his soybeans for $5.45/*bu*. In May, cash soybeans are $5.45/*bu* and the November futures price is $5.75/*bu*.

a)   Provide a reason for the November futures price being higher than the current spot price.

b)   The farmer decides to hedge. By November the cash price has declined to $4.80/*bu* and the November futures price is $5.10/*bu*. At this point the farmer lifts the hedge and sells the soybeans. What effective price did the farmer receive for his soybeans?

c)   Compute the basis in May and in November, and establish if the basis narrowed or widened. Did the basis move in a favorable direction for the farmer? Explain.

(4)   A wheat exporter receives an order in late July for 50,000 bushels of wheat to be shipped in March of the following year. The exporter does not have the wheat in

inventory and needs to purchase it before the shipping date. To lock into a price, the exporter decides to hedge using the March futures contract (which controls 50,000 bushels). The current futures price is $2.90/*bu*, and the spot price is $2.70/*bu*.

a)   Set up strategy for the exporter and analyze it under the assumption that at the time of lifting the hedge, the basis had widened by $0.15/*bu* and the futures price was $2.98/*bu*.

b)   Repeat (a) assuming the basis had narrowed by $0.15/*bu*. How does a widening or narrowing basis affect the results?

(5)   T. Knudsen Sorghum Inc. expects to harvest 1*m* cwt Sorghum in late September. The cash flows of the firm are tied solely to this product. The firm is investigating alternative ways of laying off this risk by selling futures contracts. Unfortunately, there are no liquid futures contracts on sorghum so the firm has to look at related products. Sorghum resembles corn, bith in its cultivation and in its end uses. Specifically, both products are used either as livestock food or, in a variety of processed foods for humans. Both products require the same warm temperatures and rainfall distribution. As a result, the demand and price relationships for these two products should be similar. The firm decides to investigate whether a cross hedge could be effective. The following data on the price of sorghum and on the futures price of the nearest to maturity futures contract on corn were collected over the July/August/September periods.

Day #	Sorghum Price	Corn Futures Price	Day #	Sorghum Price	Corn Futures Price
1	4.39	2.415	26	4.50	2.3875
2	4.27	2.360	27	4.45	2.3650
3	4.28	2.3525	28	4.48	2.38
4	4.32	2.3725	29	4.46	2.3655
5	4.29	2.3525	30	4.48	2.3775
6	4.37	2.4125	31	4.51	2.3675
7	4.41	2.4300	32	4.50	2.3725
8	4.39	2.4175	33	4.50	2.3675
9	4.38	2.405	34	4.41	2.33
10	4.39	2.425	35	4.45	2.34
11	4.39	2.440	36	4.43	2.3175
12	4.34	2.435	37*	4.37	2.2975
13	4.37	2.445	38	4.30	2.3375
14	4.30	2.4125	39	4.36	2.3525
15	4.29	2.390	40	4.33	2.3675
16	4.33	2.3575	41	4.27	2.33
17	4.39	2.40	42	4.28	2.345
18	4.45	2.4125	43	4.32	2.3575
19	4.41	2.41	44	4.30	2.35
20	4.38	2.3925	45	4.37	2.3875
21	4.41	2.41	46	4.32	2.3675
22	4.36	2.365	47	4.38	2.3875
23	4.48	2.3875	48	4.35	2.3850
24	4.55	2.4075	49	4.46	2.4375
25	4.50	2.3775	50	4.37	2.4425

*   On day 37, the nearest futures contract changed. Hence the change in futures price from day 36 to day 37 is not defined.

(a)   Using the data for days $1 - 25$, run an appropriate regression between daily sorghum price changes and daily changes in the corn futures contract. Provide a report on this regression and establish the hedge ratio. Based on an in-sample analysis, how effective will the hedge be?

(b)   Using the hedge ratio in (a), compute the daily error terms out of sample, and establish the effectiveness of the hedge out-of-sample.

(c)   Compare the unhedged position with the cross-hedge and draw conclusions for the firm.

(6)   Prepare a case study of a hedging problem of your own choice. Establish a hedging scenario, collect data, analyze the data, recommend a hedge.

a)   Describe a Scenario: Set up a story and suggest the futures contracts that might be considered.

b)   Collect Data: Collect your own data and perform the appropriate statistical analyses. Sources for data include the Wall Street Journal and the statistical annuals of the various futures exchanges. You probably will need spot prices as well as futures prices.

c)   Data Analysis: Estimate the risk minimizing hedge and evaluate its effectiveness in sample and out-of-sample. Make sure the assumptions of regression analyses holds. Provide an appendix with relevant computer output.

d)   Recommendations: State very precisely the hedge that you recommend and how it will meet the risk management objectives laid out in (a).

e)   Turn in an executive summary report, with your recommendations, and an appendix in which the relevant data is listed, etc.

(7)   A merchant holds an inventory of one million bushels of soybeans. The current spot price is 500c per bushel. The standard deviation of returns for soybeans is 0.20. The merchant wants to construct a risk minimizing hedge using soybean futures. Each futures contract controls 5000 bushels.

a)   If a simple 1-1 hedge was to be set up, how many contracts would the merchant sell?

b)   Assume the volatility of the futures is 0.27. For the particular grade of beans in inventory, the correlation between futures and spot is 0.80. Using this information, compute the risk minimizing hedge ratio and determine how many contracts the merchant should trade.

c)   Compare your answer in (b) and (c) and explain the cause of their differences.

(8)   A portfolio manager has a diversified portfolio worth 200 million dollars. When the returns on this portfolio are regressed against the returns on the S&P 500, the beta value is 0.91. The portfolio manager is concerned that prices will fall and would like to reduce the market risk.

a)   How can the manager use futures on the S&P 500 to reduce the market exposure to zero? Explain exactly what information is required to set up the number of futures contracts to sell. How can a regression analysis be used to reveal the potential effectiveness of the hedge.

b)   Assume the S&P 500 futures contract trades at 1000. Each contract has a multiplier of 250. Establish the number of contracts that need to be traded to rid the position of market risk.

c)   After a few days the trader decides that some exposure to the market might be appropriate. Ideally, the manager would like a beta value of 0.6. Explain how this exposure can be established using futures contracts alone.

d)   Provide some reasons why the manager would want to use futures contracts to reduce market risk, rather than portfolio reallocations?

(9)   A manager of a well diversified stock portfolio is concerned that over the next few weeks the market may decline significantly. The manager is keen to hedge against this event by trading S&P 500 futures contracts that trade at the Chicago Mercantile Exchange. The manager knows that the portfolio of stocks that he holds has a beta value of 1.20. This beta value is computed against the $S\&P$ 500 index. Assume the S&P index is at 1000, and that each futures contract has a multiplier of 250. The value of the portfolio is 10 million dollars.

a)   Given a multiplier for the futures contract is 250, establish a hedge ratio given a naive 1-1 hedge is constructed.

b)   Explain the problem with this hedge, and correct it using a preferable procedure.