Pricing Forwards and Futures

Peter Ritchken

Objectives

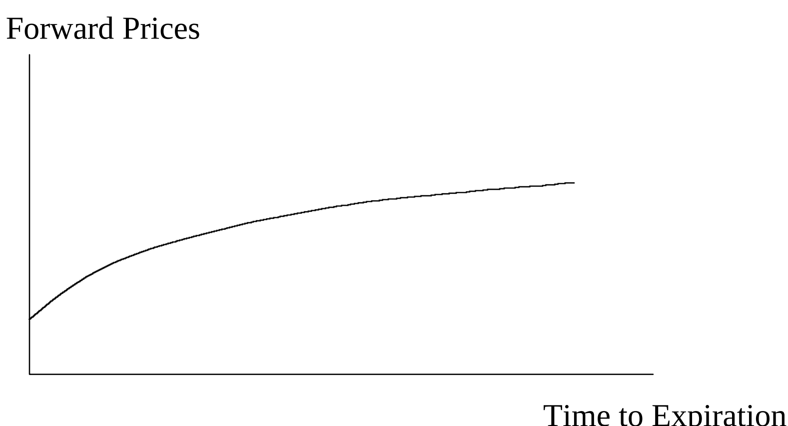
- You will learn
  - how to price a forward contract
  - how to price a futures contract
  - the relationship between futures and forward prices
  - the relationship between futures prices and expected prices in the future.
- You will use
  - arbitrage relationships
  - become familiar with the cost of carry model
  - learn how to identify mispriced contracts.

Forward Curves

- Forward Prices are linked to Current Spot prices.
  - The forward price for immediate delivery is the spot price.
  - Clearly, the forward price for delivery tomorrow should be close to todays spot price.
  - The forward price for delivery in a year may be further disconnected from the current spot price.
  - The forward price for delivery in 5 years may be even further removed from the current spot price.

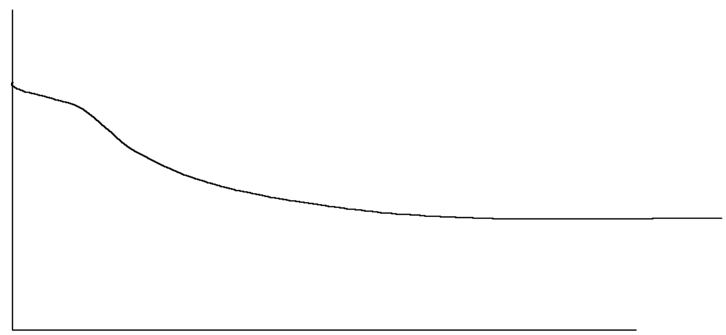
Forward Prices of West Texas Intermediate Crude Oil.

- A Contango Market



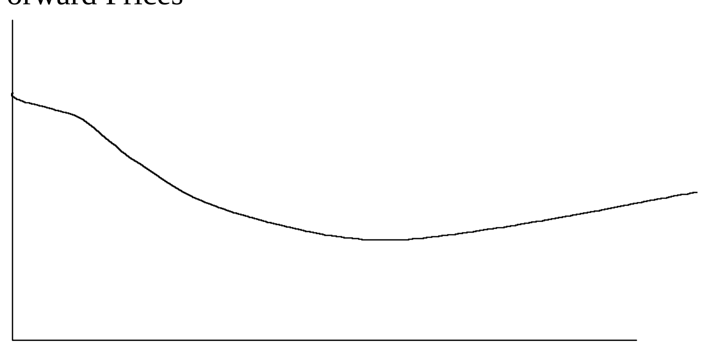
Forward Prices of West Texas Intermediate Crude Oil.

- A Backwardation Market



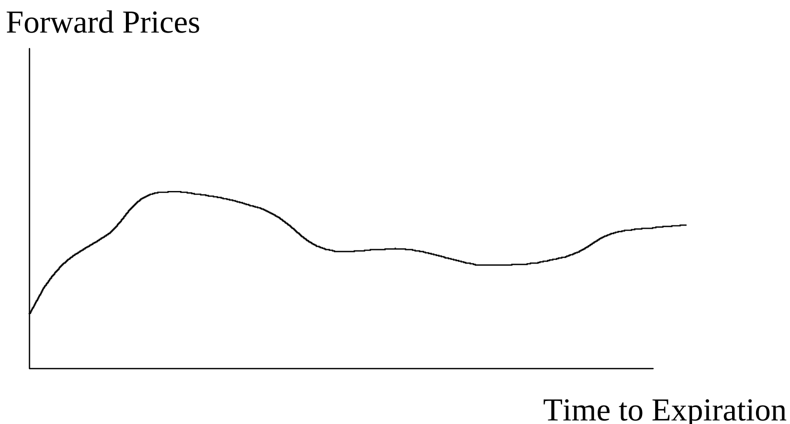
Forward Prices of West Texas Intermediate Crude Oil.

- Short term Backwardation/Long term Contango



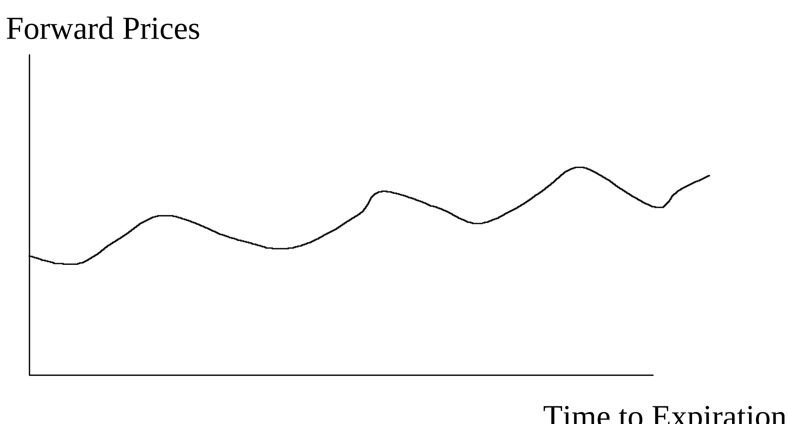
Forward Prices of West Texas Intermediate Crude Oil.

- Mixed Contango/Backwardation Forward Curve



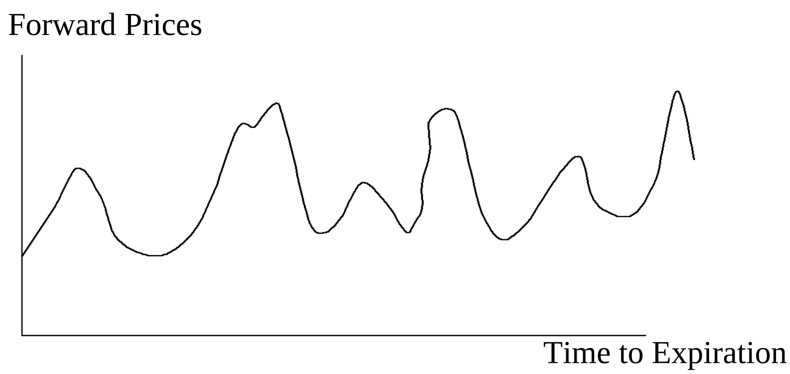
Forward Prices of Heating Oil.

- Peaks in Winter and lows in Summer.



Forward Prices of Electricity.

- Peaks in Winter and Summer with lows in winter and fall.



What determines the term structure of forward prices?

- How can we establish the fair forward price curve?
- Does the forward curve provide a window into the future?
  - Do forward prices predict future expected spot prices?
  - What can we learn from forward prices?
- Do futures prices equal forward prices?
- What can we learn from futures prices?

Forward and Futures Prices

- We make the following assumptions:
  - No delivery options.
  - Interest rates are constant.

  - This means there is only one grade to be delivered at one location at one date.
  - S(0) is the underlying price. F(0) is the forward price and T is the date for delivery.

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The Value of a Forward Contract

- At date 0: V(0) = 0
- At date T: V(T) = S(T) - FO(0)

- What about V(t)?

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Determining V(t)

	Value at 0	Value at t	Value at T
Buy Forward at date 0	0	V(t)	S(T)-F(0)
Sell a forward at date t	-	0	-(S(T)-F(t))
Value of Strategy		V(t)	F(t) – F(0)

What is V(t)?

- V(t) = Present Value of F(t) - F(0).
- BUT F(t) and F(0) are known at date t.
- Hence the payout is certain.
- Hence we have:

- $V(t) = \exp(-r(T-t))[F(t)-F(0)]$

Property

- The value of a forward contract at date t, is the change in its price, discounted by the time remaining to the settlement date.
- Futures contracts are marked to market. The value of a futures contract after being marked to market is zero.

Property

- If interest rates are certain, forward prices equal futures prices.
- Is this result surprising to you?

With One Day to Go

	Initial Value	Final Value
Long 1 forward	0	S(T) – FO(T-1)
Short 1 futures	0	-(S(T) – FU(T-1)
	0	FU(T-1)-FO(T-1)

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With Two Days to Go

	Initial Value	**Final Value**
Long 1 forward	0	[FO(T-1) – FO(T-2)]B(T-1,T)
Short B(T-1,T) futures	0	-[FU(T-1) – FU(T-2)]B(T-1,T)
	0	[FU(T-2)-FO(T-2)]B(T-1,T)

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With Three Days to Go

	Initial Value	Final Value
Long 1 forward	0	[FO(T-2) – FO(T-3)]B(T-2,T)
Short B(T-1,T) futures	0	-[FU(T-2) – FU(T-3)]B(T-2,T)
	0	[FU(T-3)-FO(T-3)]B(T-2,T)

Example: Tailing the Hedge

- Little Genius sells a Forward Contract. Then hedges this exposure by taking a long position in x otherwise identical futures contracts.
  - What should x be?
- With T years to go to expiration, the number of futures contracts to purchase is

$$x = \exp(-rT)$$

- The strategy is dynamic, since the number of futures to hold changes over time. (Actually increases to 1 as T goes to 0)

Example

- FO(0) = FU(0) = F(0) = 100
- F(1) = 120; F(2) = 150; F(3) = 160
- Profit on Forward:

  160 - 100 = 60
- Profit on Futures :

  20exp(r 2/365) +30exp(r 1/365) + 10.

Example: Now Tail the Hedge.

- With 3 days to go: Buy N_3 = exp(-r 2/365) futures.
- With 2 days to go: Buy N_2 = exp(-r 1/365) futures.
- With 1 day to go: Buy N_1 = 1 futures.
- Profit on this strategy is

$20N_3 e^{r2/365} + 30N_2 e^{r1/365} + 10 = 20 + 30 + 10 = 60$

- This is the payout of a forward.

Property

- If futures prices are positively correlated with interest rates, then futures prices will exceed forward prices.
- If futures prices are negatively correlated with interest rates, then futures prices will be lower than forward prices.

"Proof" For Positive Correlation.

- FU prices increase, the long wins and invests the proceeds at a high interest rate.
- FU prices decrease, the long looses, but finances the losses at a lower interest rate.
- Overall, the long in the futures contract has an advantage.
- The short will not like this, and will demand compensation in the form of a higher price.

Pricing of Forward Contracts

- Consider an investment asset that provides no income and has no storage costs. (Gold)
- If the forward price, relative to the spot price, got very high, perhaps you would consider buying the gold and selling forward.
- If the forward price, relative to the spot price, got very low, perhaps you would consider buying the forward, and selling the asset short!

- Lets take a closer look at the restriction these trading schemes impose on fair prices.

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Pricing Forward Contracts

- Little Genius starts off with no funds. If they buy an asset, they must do so with borrowed money. We first consider the following strategy:
  - Buy Gold, by borrowing funds. Sell a forward contract.
  - At date T, deliver the gold for the forward price. Pay back the loan.

- Profit = F(0)- S(0)exp(rT)

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Cost of Carry Model

- Clearly if F(0) > S(0)exp(rT), then Little Genius would do this strategy. Starting with nothing they lock into a profit of $F(0)-S(0)e^{rT} > 0$!
- To avoid such riskless arbitrage, the highest the forward price could go to is $S(0)e^{rT}$.
- $F(0) < S(0)e^{rT}$.

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Reverse Cash and Carry: (In a Perfect Market)

- Note that profit from the startegy is known at date 0!
- If positive, Little Genius does the strategy!
- If negative, Little Genius does the opposite!
- That is LG buys the forward contract, and sells gold short. Selling short generates income which is put into riskless assets.
- Profit = S(0)exp(rT) - F(0)

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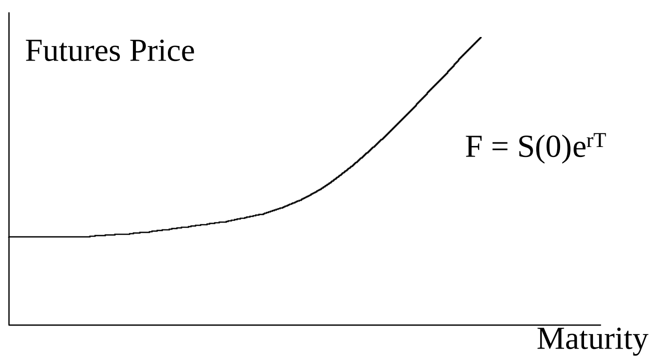
Reverse Cash and Carry: (In a Perfect Market)

- If Profit = S(0)exp(rT) - F(0) >0, Little Genius would make riskless arbitrage profits.
- Hence:
- $S(0)e^{rT}<F(0)$
- That is, to avoid riskless arbitrage, the forward price must be bigger then the future value of a riskless loan of S(0) dollars.
- Hence: $S(0)e^{rT} < F(0) < S(0)e^{rT}$,
- Or $F(0) = S(0)e^{rT}$

Term Structure of Gold Futures Prices (In a Perfect Market)

- A Contango market for Gold!



Arbitrage Restriction

- To avoid riskless arbitrage, the price of a forward contract on Gold is:

$$F(0) = S(0)\exp(rT)$$

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Example:

- S(0) = 400; F(0) = 450; T = 1 year;
  Simple Interest Rate = 10% per year.

Borrow $400 for 1 year at 10%	+400
Buy 1 ounce of Gold	-400
Sell 1 forward contract	0
Net cash flow	0

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Example
(continued)

- After 1 year:
- Remove gold from storage and deliver: 450
- Repay loan, including interest -440

F(0)

- Net Cash Flow = +$10

- Initial Investment = $0.
- To avoid arbitrage free profits from this strategy:

  F(0) < 440

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Example
Cash and Carry with Market Imperfections

- S(0) = 400; F(0) = 450;
  Simple interest rate = 10%;
  Transaction Cost = 3% of spot.

- At Date 0:
  Sell Forward Contract
  Borrow $412 at 10%.............+412
  Buy 1oz. Of Gold..................-412
  Net Cash Outflow................. 0

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Cash and Carry with Market Imperfections (continued)

- At Date T:

Remove Gold from Storage and
Deliver ....+450.00
Repay Loan............................- 453.20
Total Cash Flow......................   -3.20

- Hence,

  F(0) < $453.20

Reverse Cash and Carry with Market Imperfections

- S(0) = 400;F(0)=450;Interest Rate = 10%
  Transaction Cost = 3%

- At Date 0

Sell 1oz. gold short
Receive 400(0.97) $388
Invest proceeds at riskless rate.......-$388
Net Cash Flow..................................$0

Reverse Cash and Carry with Market Imperfections (continued)

- At Date T:

Accept loan proceeds
[388(1.1)]......................426.80
Accept gold delivery....-450.00
Total Cash Flow............-23.20

- Therefore, you would have arbitrage free profits if

  F(0) < $426.80

Arbitrage Free Bounds

- Hence, to avoid riskless arbitrage:

  426.8<F(0)<453.2

- The size of the bounds increase with market imperfections.
- However, the actual size of the bounds are determined by the market players that face the least imperfections!

Pricing Futures Contracts on Stocks

- S(0) = 100
- d = $2 in 0.5 years.
- Interest = 10% simple.
- Forward contract is for 1 year
- At Date 0
  - Forward contract for 1 year.
  - Borrow $100 ....+100
  - and Buy stock............-100

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Pricing Futures Contracts on Stocks

- At Date t=0.5
  - Receive $2.
  - Invest $2 at 10% per year
- At date T= 1
  - Collect proceeds from dividend........$2.10
  - Sell stock for forward price................F(0)
  - Repay loan.......................................-$110
  - Profit = F(0)-110+2.10
  - Profit is positive if F(0) exceeds 108.90

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Futures Prices on Stock

To avoid arbitrage opportunities the forward price is bounded by :

$$F(0) \le S(0)e^{rT} - de^{r(T-t)}$$
$$F(0) \le S(0)[1 + rT] - d[1 + r(T - t)]$$

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Forward Prices on a Stock

- **To avoid arbitrage opportunities the forward price of a stock is**

- **F(0) = S(0)exp(rT) - d exp(r(T-t))**
- **F(0) = S(0)[1+rT] - d[1+r(T-t)]**

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Futures Price on a Nondividend Paying Stock

- Does the futures curve provide any forecasting power for the future stock price?
- If the slope of the futures curve increases
  - The prospects for the stock has improved?
  - Interest Rates have increased?

Stock Market Indices

	Time 0	Time t	Shares Outstanding
A	150	150	50
B	40	80	100
C	10	30	500

Market Value Indices

- MV(0) = 150(50)+40(100)+10(500)=16500
- MV(t) = 150(50)+80(100)+30(500)=30500

- I(0) = 100
- I(t) = I(0)[MV(t)/MV(0)]
  =100[30500/16500] = 184.85

Price Weighted Index

- V(0) = [150+40+10]/3 = 66.667
- V(t) = [150+80+30]/3 = 86.67

- I(t) = [V(t)/V(0)]I(0)
  = [86.67/66.667]100
  =130

Pricing Futures on Price Weighted Indices

- Stocks in the index are A and B.
- A pays a dividend of size d_A at time t_{A}.
- B pays a dividend of size d_B at time t_B.

$$F(0) = S_A(0)e^{rT} - d_A e^{r(T-t_A)} + S_B(0)e^{rT} - d_B e^{r(T-t_B)}$$

$$F(0) = [S_A(0) + S_B(0)]e^{rT} - [d_A e^{r(T-t_A)} + d_B e^{r(T-t_B)}]$$

$$F(0) = I(0)e^{rT} - FV(dividends)$$

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Stock Index Arbitrage with a Value Weighted Index

	Price	Shares	Div	Time to Div.
A	40	1m	0.5	10 days
B	35	2m	0.5	12 days
C	25	2m	-	-

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Example:
Stock Index Arbitrage with Price Weighted Index

- Composition of Index:
  - A= 40/ (40+35+25) = 1/4
  - B = 7/16
  - C=5/16
- I(0) = 400
- Multiplier = 500
- Each contract controls 400(500) = $200,000.

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Stock Index Arbitrage

- Portfolio Composition:
  - (1/4)(200,000) = $50,000 or 1,250 shares of A
  - (7/16)(200,000) = $87,500 or 2,500 shares of B
  - (5/16)(200,000) = $62,500 or 2,500 shares of C

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Stock Index Arbitrage

Stock	A	B	C
Borrowed Funds	50,000	87,500	62,500
Number Shares	1,250	2,500	2,500
Dividend Income Received and Invested at 10%	628.43	1256.18	-

Stock Index Arbitrage

- Amount owed = 200,000exp(0.10(30/365)
  = $201,650.61
- less dividends and interest............. -$1,844.61.
- Total = ...$199,766
- Theoretical Futures Price = 199,766/500
  = 399.53

Note, in this example I(0) = 400

Stock Index Futures Prices and Index Prices

- If Stock index futures prices are upward sloping as a function of maturity, then market participants are forecasting that the market will increase (ie the sentiment is bullish)
- Is this true?

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Forward Contracts on FOREX

- Price of 1 British Pound is S(0) dollars.
- Strategy 1:
  - Invest S(0) dollars at the risk free rate to obtain
  - $S(0)\exp(r_D T)$ dollars at date T.
- S(0) dollars grows to $S(0)\exp(r_D T)$ dollars

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Forward Contracts on FOREX

- Strategy 2:
  - Sell $\exp(r_F T)$ forward contracts on the pound.
    Each contract gives the obligation to exchange 1 pound for F(0) dollars.
  - Buy 1British pound. Invest in riskless rate in Britain to obtain $\exp(r_F T)$ pounds.
  - Deliver the pounds for $\exp(r_F T)$ F(0) dollars.
- S(0) grows to $\exp(r_F T)$ F(0)

Forward Contracts on FOREX

- To avoid arbitrage opportunities:
- $S(0)\exp(r_D T) = \exp(r_F T) F(0)$

or

$$F(0) = S(0)\exp[(r_F - r_D)T]$$

- Forward prices relate to spot prices depending on interest rate differentials.

Pricing Futures Contracts on Dividend Paying "Stocks"

- Assume interest rate is r and storage costs are a fixed percent of the spot price. The underlying pays no dividends.
- F(0) = S(0)exp[(r+u)T]
- Now assume the underlying pays a continuous dividend yield, expressed as a percent of the price. Then
- F(0) = S(0)exp[(r+u-d)T]

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Pricing Futures Contracts on Dividend Paying "Stocks"

- F(0) = S(0)exp[(r+u-d)T]
- The spot price of the S&P500 index is 1000. The dividend yield is 3% per year. Interest rates are 5% continuously compounded. Storage costs are 0%. A one year futures contract should have a futures price of
- Then $F(0) = S(0)e^{(r-d)T} = 1000\ e^{0.02} = 1020.20$

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Forward Prices of a Property

- You can buy a property now for $100m. Alternatively, you can enter into a forward contract to purchase the property in two years time.
  - The property has a maintenance fee that is 1% of the price. The property has buildings that provide rental income that is estimated at 6% of the price per year.
- The fair forward price of the property is

  $100e^{(r+u-d)T} = 100e^{(0.05+0.01-0.06)2} = 100$ million dollars.

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Futures Prices of Storable Commodities.

- Commodity forward contracts have two important features that are not present when the underlying is a financial asset.
  - **Storage Costs**
  - **Convenience Yields**
- **Storage Costs**
  - **Warehouse space, transportation costs, spoilage, insurance.**
  - **We will represent these charges as a fraction of the market price of the commodity.**
  - **If storage costs are 20%, this implies that the annualized storage costs are about 20% of the spot market price.**

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Futures Prices of Storable Commodities.

- **The convenience yield**
  - Unlike securities, commodities are usually consumed or used in a production process.
  - Having a commodity on hand has value since it allows the production process to continue without disruptions.
  - If the commodity is abundant then there is not much benefit from having it in storage. However,
  - If the commodity is scarce, then having the commodity in inventory is very beneficial.
- One can view the potential benefit of having a commodity on hand as a yield, just like a continuous dividend yield. We express the convenience yield as a percent of the price in an annualized form

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Futures Prices of Storable Commodities.

- A 2% convenience yield means that having the item readily available comes at a cost that accrues at a rate of 2% of the current price of the commodity per year.

Example:

Spot price = 100; Interest Rate = 5%; Storage Cost = 8%

Convenience Yield = 7%. Time to expiration is 1 year.

$F(0) = S(0)\, e^{(r+u-k)T} = 100\, e^{(0.05+0.08-0.07)} = 106.18$

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Upper Bound for Commodity Forwards: The Cost of Carry Model

- If forward price is very high:
  - Sell the forward contract
  - Buy the commodity using borrowed funds.
  - Pay the storage fees with borrowed funds
  - Deliver the commodity for the forward price.
- Profit is:

  $FO(0) - S(0)e^{(r+u)T}$

  Clearly, to avoid riskless arbitrage:

  $F0(0) < S(0)e^{(r+u)T}$

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Lower Bound for Commodity Forwards

- If the forward price was very low we would like to initiate the reverse cost of carry. In particular, we need to short sell the commodity.
- We must borrow the commodity from party, A, and then sell it.
- This deprives A from a convenience yield. A is giving up having inventory around to meet unexpected needs! Clearly A needs to be compensated for this.
- This compensation is the convenience yield, represented by k% per annum as a fraction of the commodity price

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Lower Bound for Commodity Forwards

- A does get to save the storage fees, represented by u% per annum as a fraction of the commodity price.
- So A requires a net compensation that accrues at the rate of k-u%
- If we borrow one unit now, we have to compensate A by providing $e^{(k-u)T}$ units at date T.
- Equivalently, for every $e^{-(k-u)T}$ units that we take today we owe 1 unit at date T.

Lower Bound for Commodity Forwards

- So we go long 1 forward contract, and we borrow $e^{-(k-u)T}$ units, and sell them.
- At date T, we purchase 1 unit for the forward price, and return 1 unit to the borrower. The net profit is:

  Profit $= [e^{-(k-u)T}]S(0)e^{rT} - FO(0)$

  $= S(0)e^{(r+u-k)T} - FO(0)$

- Profit if $S(0)e^{(r+u-k)T} > FO(0)$
- Hence to avoid riskless arbitrage:

- $FO(0) > S(0)e^{(r+u-k)T}$

Commodity Forward Prices.

$S(0)e^{(r+u-k)T} < FO(0) < S(0)e^{(r+u)T}$

- The upper bound is easy to obtain. The lower bound is a problem, since the convenience yield is not known.
- Forward prices on commodities can have such wide upper and lower bounds, that little can be said about these prices from arbitrage arguments alone!
- We need a deeper model.

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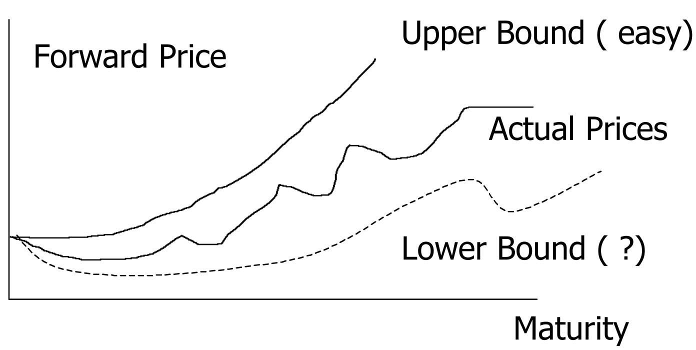
Bounds on Forward Prices



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Interpreting the Convenience Yield

- Let k be the convenience yield. You can view this like a dividend yield, but it is not observable!
- Then, we will write:
  $F(0) = S(0)\exp[(r+u-k)T]$
- When will k be large?
- When will k be small?
- Can k depend on the time horizon? (ie. Different k values for different futures contracts.)

The Implied Convenience Yield

- If the convenience yield was k%, then
  $FO(0) = S(0)e^{(r+u-k)T}$
  - In practice the forward prices are given. So we can imply out the average convenience yield over the period [0,T].

Example
On April 1st, $S(0) = 1.96$, $u = 1\%$, $r=9\%$

Settle Date	May	July	Sept	Dec	March
Futures Price	1.95	1.92	1.87	1.89	1.89
Upper Bound	1.976	2.001	2.043	2.095	2.148
Implied k	16.13%	18.24%	21.28%	15.46%	13.96%

Implied Convenience Yields

- Consider maturity s and t contracts with s<t
- $FO_t(0) = FO_s(0)e^{(r+u-k)(t-s)}$
- Why is this true?
- We can imply out convenience yields over specific periods.

Example (Continued)
On April 1st, S(0) = 1.96, u = 1%, r=9%

Settle Date	May	July	Sept	Dec	March
Futures Price	1.95	1.92	1.87	1.89	1.89
Upper Bound	1.976	2.001	2.043	2.095	2.148
Implied k	16.13%	18.24%	21.28%	15.46%	13.96%
Implied rates	April-May 16.13%	May –July 19.29%	July-Sept 25.83%	Sept-Dec 5.74%	Dec-Mar 10%

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Forward Prices, Spot Prices, and Forecasts

- Expect future prices to be high:
  - Sellers will increase their inventories so as to capture the high prices.
  - Buyers will want to buy more today.
  - Buyers will demand more from sellers who prefer to sell less.
  - These conflicting positions will drive spot prices up
  - Price expectations are an important part of the formation of spot prices.
- With inventory, changes in future expectations have impact on current prices.

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Price Variations and Storage Issues

- As storage costs increase, the connections between expected future prices and current prices diminish.
  - With high storage costs the alternative of holding inventories for future sale is less attractive than if storage costs are negligible.
  - As storage costs increase, expectations of higher future prices will have less impact on current spot prices.
  - As storage costs increase, the price variation over time in the forecasts increase.
  - As storage costs increase, actual price variations over time increase.

Example: Electricity

- Electricity is difficult to store. Expected price forecasts vary by the hour.
- Actual prices fluctuate widely.
- Cost of carry model and reverse cost of carry model do not hold.

Spot Forecasts Versus Forward Prices

- What is the link between forward prices and expected spot prices for non storables?

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Futures Prices and Expected Future Spot Prices

- Suppose k is the expected return required by investors on an asset. (k = r + risk premium)
- We could invest Fexp(-rT) today to get S(T) at date T.
  - How would you do this?
- Hence

$$Fexp(-rT) = E[S(T)]exp[-kT]$$

- No systematic risk?
- Positive systematic risk?
- Negative systematic risk?

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Futures Price For Nonstorable Commodities

- $E[S(T)] = 100$
- $F(0) = 100$
- Perhaps a speculator would enter a forward contract at 90. Expect to make a $10 profit.
  - What if $F(0) = 99$
  - The speculator expects to make $1, but risk….
- Futures prices may not be unbiased estimators of future spot prices.

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Normal Backwardation

- If all hedgers were SHORT, all speculators, LONG.
  - Hedgers will pay a premium to reduce risk
  - Speculators will require a premium to accept risk.
  - $F(0) < E[S(T)]$
- This market is called normal backwardation.

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A Contango Market

- If all hedgers were LONG, all speculators, SHORT
- Hedgers will pay a premium to reduce risk
  - Speculators will require a premium to accept risk.
  - F(0) > E[S(T)]
- This market is known as CONTANGO.

- If F(0) = E[S(T)]= E[F(T)]….
  - Futures prices are martingales

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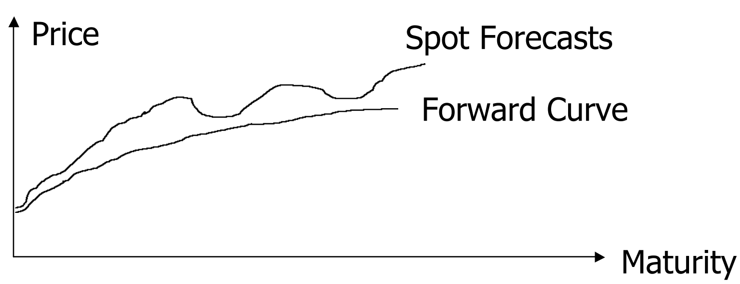
Expected Spot Prices and Forward Prices

- The risk premium equals the difference in prices (expected spot less forward), as a percentage of the forward.



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Risk Premiums
Producers Perspective

- There is no theoretical reason for forward prices to act as an unbiased estimator of future expected prices.
- The risk premium may be positive or negative!
- The risk premium may start out big positive, say, and then decline to zero at the expiration date.
- The risk premium exists due to risk aversion. Producers (sellers) are generally risk averse. They may have debt obligations associated with their capital stock. They need stable cash flows.
- So, they may be willing to accept forward prices that are less than expected spot prices.

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Risk Premiums
Consumers Perspective

- Buyers may also have a desire to avoid price risk.
- They may prefer to pay a premium for price security. This implies that forwards might be higher than expected prices and risk premiums would be negative.
- Relative desire for price security between these groups that will determine the sign of the risk premium.

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Speculators

- Speculators are also present. They are concerned about forward prices relative to expected future prices.
- Does the strategy of buying forward and then selling at spot prices increase the total risks of a well diversified portfolio?
  - Yes-speculators need a positive risk premium. Forward will be below expected prices

The Market Price of Risk

- The Risk Premium can be decomposed into two parts:
  - Volatility (or the amount of risk)
  - The price per unit of risk.
- The price per unit of risk is called the market price of risk.
- Estimating the market price of electricity risk is difficult.

Summary

- Forward prices and futures prices are close to each other.
- Futures give cash flows right away, forward do not. This has hedging implications.
- For securities, the cost of carry model and the reverse cost of carry model gives us forward prices.
- Market imperfections lead to bounds on prices.
- For commodities storage costs and convenience yields must be considered.
- The upper bound is easily computable, but the lower bound depends on the convenience yield.

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Summary

- For commodity forwards and futures there is some flexibility as to where the forward prices can fall.
- That is, arbitrage alone, cannot identify the theoretical price, and we need richer models.
- When storage costs are high, forward prices and indeed spot prices and their forecasts can be more volatile. Cheap storage has the effects of smoothing prices over time.
- Implied convenience yields can be extracted from futures prices and can be informative.

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Summary

- In general commodity forward curves will reflect seasonality since the convenience yields will fluctuate according to aggregate inventory levels.
- Electricity is not readily storable, and forward prices here represent a challenge.
- For non storables, forward prices are linked to expected future spot prices. However, they are not unbiased estimators.

Summary

- For electricity prices, we may expect spikes and other "weird" phenomenon. For storables, as future expectations change, a ripple effect is felt through to current prices. Price changes should therefore be more smooth.
- If storage costs increase, then price volatility should increase.