Slide 1

The Binomial Model

The primary objectives are

- To investigate specific dynamic trading strategies that produce payouts equivalent to those of an option;
- To describe the binomial option pricing model; and
- To introduce the Black Scholes pricing model.

Slide 2

Dynamic Self Financing Trading Strategies

The trading strategies that we shall consider have two major restrictions.

- the strategy can only use information available at the current time.
- the trading strategy must be self financing.
- That is, the only adjustments that can be made to the composition of a portfolio are those that leave its total value unchanged.

Slide 3

- If, for example, additional shares are to be purchased, then the funds for these shares must be obtained by borrowing.
- If shares are sold, then the funds must either be used to reduce outstanding loans or must be invested at the riskless rate.
- No external funds can be injected into the trading strategy, nor can funds be removed from the trading strategy over the time period.

Slide 4

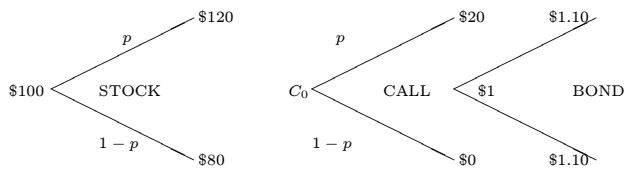
- The goal of this chapter is to identify a trading strategy involving stocks and bonds that produces the same cash flows as an European call option.
- If this could be accomplished, then, by the law of one price, the cost of the option should equal the initial cost of the particular portfolio that duplicates the option's payouts, when the self-financing strategy is followed.

Slide 5

Example

- A stock is priced at \$100. Assume that over a period the stock can either rise to 120 or fall to 80. Also assume that a one period call option with strike 100 is available. Finally, assume that money can be borrowed or lent at a 10% rate, and the stock and call can be bought or sold at the given prices.

Figure 1: Price Process for a Stock, Call and Bond.



Slide 6

What is the fair price of the call?

Slide 7

The Usual Valuation Procedure

- First compute the expected cash flows associated with the investment.
- Second, discount these cash flows at the approprite risk adjusted rate.

Slide 8

Problem 1 : Expectations or Probabilities

- We cannot compute the expected value! (why?)
- We are NOT given the probability of an up jump!
- If we were, given p, then we could compute the expected terminal value.
- If p = 0.5, then Expected Value = $20 \times p + 0 \times (1 - p) = 20p = 10$.

Slide 9

Problem 2: Discount Rates

- Now we need the discount rate. This is a big problem!
- To establish this rate requires understanding more about the preferences of investors in the economy, or at least having some notion of how the inclusion of a call option into a well diversified portfolio affects its overall risk.
- We know the risk free rate.
- Lets add a premium to the risk free rate.
- Value today = Expected Value/ $(1+r_f + \pi)$
- Value = 10p/$(1+r_f + \pi)$
- IN ORDER TO VALUE THIS CLAIM WE NEED TO KNOW P AND π.
- It appears we do not have enough information!!

Slide 10

GIVE ME A PRICE

Slide 11

- The value of the claim can be computed without explicitly knowing p,

  and
- without explicitly knowing the true discount rate for investments of equivalent risk to the claim!

Slide 12

- Indeed, given the information, the fair price of the option must be \$13.64.
- If the market sets the price of the call option at any other value, a riskless arbitrage opportunity exists.

Slide 13

I can prove this result to you!

- Consider a portfolio containing 1 stock and a short position in 2 calls.

  (How we come up with this trading strategy involving the sale of 2 call options will be discussed later. For the moment take it as a given!)

- The initial value of this position, V_0, is

$$V_0 = 100 - 2C_0.$$

  If the stock price rises, the portfolio value, V_1, will be given by

$$V_1 = 120 - 2(20) = 80$$

  and if the stock price falls, the portfolio value will be given by

$$V_1 = 80 - 2(0) = 80.$$

Slide 14

Hence, regardless of what occurs in the future, the terminal value of this portfolio is \$80.

Slide 15

- Now consider placing \$72.73 into the riskless asset (bank). At the end of the period, this wealth would have grown to $\$72.73(1.10) = \$80$.
- We thus have two riskless alternatives. Either we can buy 1 stock and sell 2 calls, or we can put \$72.73 in the bank.
- To avoid riskless arbitrage opportunities it must follow that the current values of these portfolios must be equal.
- Hence, $V_0 = 100 - 2C_0 = 72.73$ or, $C_0 = \$13.64$.

Slide 16

- Say the call was \$12.64, then by selling the portfolio, the investor would receive $100 - 2(12.64) = \$74.73$.
- By investing \$72.73 of this in the riskless asset, the investor would have \$2 left over.
- At the expiration date the value of the riskless asset would be $72.73(1.10) = \$80$, and the value of the portfolio would also be \$80.
- Thus, regardless of what occurred in the future, the \$80 received from the riskless asset would always be sufficiently large to cover the value of the portfolio.
- Hence, regardless of what occurred, the investor would make \$2 without requiring any initial investment.
- Since many arbitragers would attempt this strategy, the price of the option would soon rise to \$13.64. With the option at this value, no free lunch would exist.

Slide 17

- How did I do this.
- How did I construct a portfolio that would give me the same payout as the claim.
- ARE YOU READY TO RUMBLE!!!

Slide 18

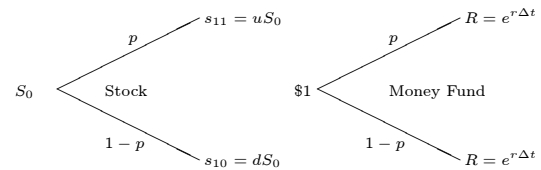
The Single Period Binomial Model

- Consider a stock currently priced at S_0.
- We assume that at the end of the period, which coincides with the expiration date, the stock price, $\tilde{S}_1$, can either increase to s_{11} or decrease to s_{10}.
- Let Δt represent the time period in years, and let r be the annualized continuously compounded rikless interest rate. A \$1.0 investment in a money fund, grows to $\$e^{r\Delta t}$ over the time period Δt.
- A call option with strike X and maturity Δt is priced at C_0. At expiration the call value, will be c_{11} or c_{10} where

$$\begin{aligned} c_{11} &= Max(s_{11} - X, 0) \\ c_{10} &= Max(s_{10} - X, 0) \end{aligned}$$

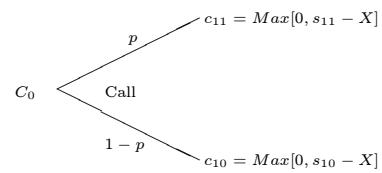
Slide 19

Figure 2: Price Processes for Stock and Money Fund.



Slide 20

Figure 3: Price Processes for Call.



Slide 21

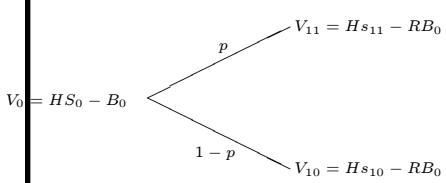
- To avoid arbitrage opportunities, we shall assume that $d < R < u$.
- Consider a portfolio containing H shares of stock that are partially financed by borrowing B dollars at the riskless rate The current value of the portfolio, V_0, is

$$V_0 = HS_0 - B_0$$

At the expiration date the portfolio value is

Slide 22

Figure 4: Portfolio Value



Slide 23

- We shall select the number of shares H, and dollars borrowed, B_0, such that the terminal values V_{11} and V_{10} are exactly equal to the call values c_{11} and c_{10}.
- That is, we require

$$\begin{aligned} Hs_{11} - RB_0 &= c_{11} \\ Hs_{10} - RB_0 &= c_{10}. \end{aligned}$$

- Since there are two equations in two unknowns, a unique solution for H and B can be obtained.
- The solution is H^*, and B^* given by some equations
- A portfolio containing H^* shares partially financed by borrowing B^* dollars produces identical payoffs to the call option at the expiration.

Slide 24

The price of a call option option with one period to go is:

$$C^* = H^* S_0 - B^* \tag{1}$$

where

$$\begin{aligned} H^* &= \frac{c_{11} - c_{10}}{s_{11} - s_{10}} \\ B^* &= \frac{c_{11}s_{10} - c_{10}s_{11}}{(s_{11} - s_{10})R} \end{aligned}$$

Slide 25

An Alternative Expression for the Call Price

- The call pricing equation, $C_0 = H^* S_0 - B_0^*$, can be simplified further.
- By substituting the value of $H^*$and B_0^* into the equation, we obtain

The price of a call option option with one period to go is:

$$C_0 = \frac{1}{R}[\theta c_{11} + (1 - \theta c_{10}] \tag{2}$$

where

$$\theta = \frac{R - d}{u - d}$$

Slide 26

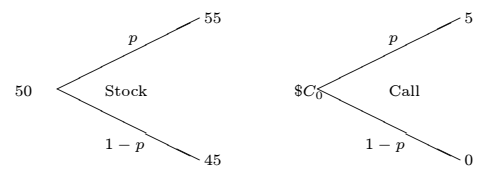
- The one period binomial call price can be viewed as the expected terminal value of the call price, discounted at the riskless rate of return, where the probability of an upward movement is given by θ.

  Example

- S(0) = \$50. A one-period call option with strike price 50 is to be valued. The stock can either increase 10 percent to \$55 or decrease 10 percent to \$45. Each one dollar investment in the money fund grows to \$1.05 at the expiration date. The stock and call prices are shown in Figure 5.

Slide 27

Figure 5: Stock and Call Prices



From the information we have $u = 1.1, d = 0.9, R = 1.05$, and $S_{11} = 55, S_{10} = 45, C_{11} = 5$, and $C_{10} = 0$.

Slide 28

Hence,

$$\begin{aligned} H^* &= \frac{5 - 0}{55 - 45} = 0.5 \\ B^* &= \frac{5(45) - 0(55)}{1.05(55 - 45)} = \$21.40 \end{aligned}$$

- A portfolio containing 0.5 shares held long partially financed by borrowing \$21.40 is the replicating portfolio. Its current value V_0 is

$$V_0 = H^* S_0 - B_0^* = \$3.60.$$

- After one period this portfolio will either increase to V_{11} or decrease to V_{10}:

$$\begin{aligned} V_{11} &= H^* s_{11} - RB_0^* = \$5 \\ V_{10} &= H^* s_{10} - RB_0^* = \$0. \end{aligned}$$

- To avoid riskless arbitrage opportunities between the call and

Slide 29

replicating portfolio, it must follow that

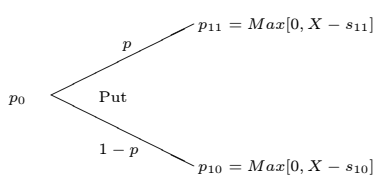
$$C_0^* = H^* S_0 - B_0^* = \$3.60.$$

Slide 30

One-Period Model for Put Options

The same procedure can be used for put options. Let P_0 be the price of the put option with strike X, and let p_{11} and p_{10} represent the terminal prices.

Figure 6: Price Process for Put



Slide 31

- Consider a portfolio of H_p shares financed by borrowing B_p. Let V_0 be the current value, i.e., $V_0 = H_p S_0 - B_p$, and let V_{11} and V_{10} be the two possible terminal values of the portfolio.
- In order for the portfolio to replicate the payoffs of the put, we must have

$$\begin{aligned} H_p^* &= \frac{(p_{11} - p_{10})}{(s_{11} - s_{10})} \\ B_p^* &= \frac{p_{11} s_{10} - p_{10} s_{11}}{(s_{11} - s_{10}) R} \end{aligned}$$

and to avoid riskless arbitrage opportunities, it must follow that

$$P_0 = H_p^* S_0 - B_p^*$$

Slide 32

Example

$S_0 = 50, u = 1.1, d = 0.9, R = 1.05$. Assume a put option is traded with strike $X = 50$. Then:

$$\begin{aligned} H_p^* &= -0.5 \\ B_p^* &= -\$26.2 \end{aligned}$$

That is, the put option can be replicated by selling 0.5 shares and lending \$26.20. Hence,

$$P_0 = H_p^* S_0 - B_p^* = -0.5(50) + 26.2 = \$1.2.$$

Slide 33

Risk Neutral Valuation

- We have seen that the fair call price does not depend on the probability of an upward jump, p.
- In light of the fact that probabilities appear unimportant, it must follow that expectations do not enter the analysis.
- However, variances cannot be ignored. The variance is a measure of spread, and this was clearly an important consideration. As u and d change, the call price changes.
- At first glance it may appear surprising that probabilities are unimportant.
- WHY IS THIS THE CASE?
- Also preferences were never used. We do not need to establish a special discount rate!

Slide 34

- WHY IS THIS THE CASE?

Slide 35

Call Prices in a Risk-Neutral Economy

- Let us consider the following experiment.
- In a risk-neutral economy, decision makers base their decisions solely on expectations without regard to the shape of the probability distribution of outcomes.
- Measures of uncertainty, captured by the standard deviation (or volatility), and the nature of higher moments do not influence decisions.
- As a result investors are indifferent between a gamble, G, that has an expected return, $E(G)$, and a certain riskless return that guarantees a payout of $E(G)$.
- Since investors are not compensated according to the size of risk, the prices of all securities will be set to yield the same riskless rate.

Slide 36

- DOES THIS SEEM REASONABLE?

Slide 37

Call Prices in a Risk-Neutral Economy

Under risk neutrality:

$$\begin{aligned} B_1 &= RB_0 \\ E(S_1) &= RS_0 \\ E(C_1) &= RC_0 \end{aligned}$$

Now, in this economy, the expected stock and call prices on the binomial lattice are

$$\begin{aligned} E(S_1) &= ps_{11} + (1-p)s_{10} \\ E(C_1) &= pc_{11} + (1-p)c_{10} \end{aligned}$$

Slide 38

where p is the probability of the stock price moving up. Hence,

$$RS_0 = ps_{11} + (1-p)s_{10}$$

or, equivalently,

$$RS_0 = puS_0 + (1-p)dS_0$$

Solving the expression for p, we obtain

$$p = \frac{(R-d)}{(u-d)} = \theta$$

Since the expected return on the call is the risk free rate we also have:

$$RC_0 = pc_{11} + (1-p)c_{10}$$

and hence

$$C_0 = \frac{[\theta c_{11} + (1-\theta)c_{10}]}{R}$$

Slide 39

- But this equation is exactly the one period binomial call pricing equation we obtained earlier.
- This tells us that the call price can be computed as the expected terminal payout discounted at the risk free rate, where the probability of an upmove is the risk neutral probability, θ. That is:

$$C_0 = E_0(C_1)/R$$

Slide 40

If somehow we knew that the fair value did not depend on preferences, then, to value an option we could assume a risk neutral economy, derive the equilibrium call price in this economy (by simply computing the present value of the expected terminal call price, and discounting at the riskless rate). The resulting call price obtained in this economy is also the fair value for our risk averse economy.

Slide 41

More on Risk Neutral Valuation

- Normally when a risky claim is to be valued, a present value calculation involving two steps is performed. First, uncertain future cash flows are replaced by their expected values and then treated as given. Second, these future expected values are discounted typically at some constant rate that reflects the nondiversifiable risk.
- Here we see that the value of an option on a binomial lattice can be established by readjusting the probability mass such that risk is reallocated in a way that allows the appropriate discount rate to be the riskless rate.
- The redistribution of probability mass, in this case, involves replacing the true probability of an upmove, p, by the risk neutral probability θ.

Slide 42

- This redistribution of probability, together with the use of the riskless discount rate, results in the option price being set at its fair value, the fair value representing the price at which investors would be prepared to pay for bearing equivalent risks.
- This process is referred to as risk neutral valuation. The name is a bit misleading since the prices obtained are fair prices in a risk averse economy. They are not prices which would only exist if investors were neutral to risk.
- The risk neutral valuation equation can only be used if an appropriate mechanism for redistributing the probability distribution can be established. Such a redistribution can always be obtained if a self-financing dynamic trading strategy can be constructed to replicate the payouts of the call.

Slide 43

The Risk-Free Hedge

- Since $C_0 = H^* S_0 - B_0^*$, it follows that the amount borrowed, B_0^*, is $H^* S_0 - C_0$.
- Equivalently, a long position of H^* shares of stock and a short position in the call, must produce a return equivalent to that of an investment in the riskless asset.
- As an example, consider our illustrative problem. We shall construct a portfolio containing a long position of 0.5 shares (H^*shares) and a short position of 1 call option. The current value of this portfolio is

$$V_0 = 0.5 S_0 - C_0 = \$21.40.$$

Slide 44

- The terminal value of this portfolio is either V_{11} or V_{10} where

$$\begin{aligned} V_{11} &= 0.5 s_{11} - c_{11} = 0.5(55) - 5 = 22.5 \\ V_{10} &= 0.5 s_{10} - c_{10} = 0.5(45) - 0 = 22.5 \end{aligned}$$

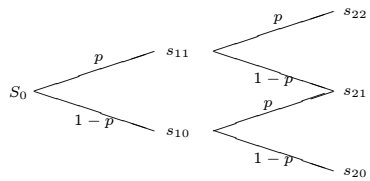
- Hence, regardless of what stock price occurs in the future, the portfolio value is known (\$22.50).
- Thus, a bond may be replicated by buying H^* shares and selling 1 call. The ratio of shares bought per call sold, H^*, is called the *hedge ratio*.

Slide 45

The Two-Period Binomial Model

- We now extend the one-period model to a two-period model where an opportunity exists to revise the position at the beginning of each period. The stock price movements are :

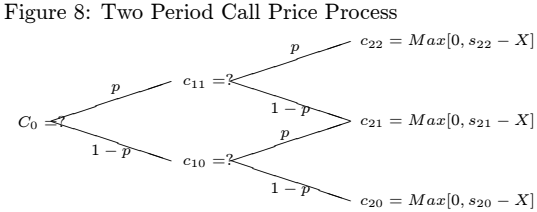
Figure 7: Two Period Stock Price Process.



Slide 46

- The call price movements can be represented as

Figure 8: Two Period Call Price Process



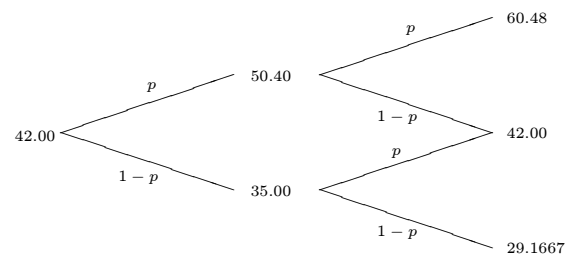
Slide 47

- With one period to go to expiration, the stock price is either at s_{11} or s_{10}. —item If the stock price is at s_{11}, then, we are faced with a one-period problem, and fair call value c_{11} can be obtained.
- Similarly, if the stock price is at s_{10}, the theoretical call value c_{10} can be obtained.

  Example
- Consider a stock currently priced at \$42.00. Assume $u = 1.20$ and $d = 1/u = 0.83333$ and $R = 1.05$ per period. Consider a call option with strike $X = \$40$. The stock price can be represented as

Slide 48

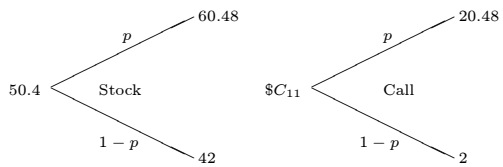
Figure 9: Stock Price Process



Slide 49

Case 1. Assume the stock price goes to \$50.4. Then, in the last period we have:

Figure 10: Prices in the Last Period Given the Stock Moves Up in the First Period



- $H^* = (20.48 - 2)/(60.48 - 42) = 1, B^* = (20.48(42) - 2(60.48))/[(60.48 - 42)(1.05)] = \$38.0952$ and $c^*_{11} = H^* s_{11} - B^*_0 = \$12.30476$.

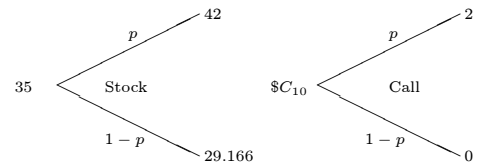
Slide 50

- Thus, at the beginning of period 1, if the stock price is \$50.4 and if the call price is not \$12.304, riskless arbitrage opportunities exist.

Slide 51

Case 2. Assume the stock price in period 1 is \$35:

Figure 11: Prices in the Last Period Given the Stock Moves Down in the First Period

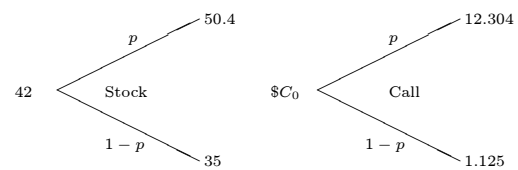


- $H^* = (2 - 0)/(42 - 29.166) = 0.155844$ and $B^*_0 = (2)(42)/[(42 - 29.166)(1.05)] = 23.1255$. Thus,$c^*_{10} = (0.155844)(35) - 23.1255 = 1.1255$.

Slide 52

If we knew with certainty that at the end of the first period the call price would either be c^*_{11} or c^*_{10}, then to obtain the current fair value we could reapply the one-period model.

Figure 12: Prices Over the First Period



- The replicating portfolio consists of

$$H^* = (12.304 - 1.1255)/(50.4 - 35) = 0.7259$$

Slide 53

$$B^* = [12.304(35) - 1.125(50.4)]/[(50.4 - 35)(1.05)] = \$23.125$$

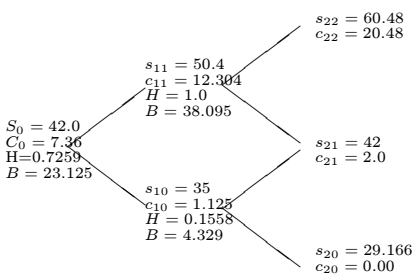
and hence

$$C_0^* = H^* S_0 - B_0^* = (0.7259)(42) - 23.125 = \$7.36.$$

Slide 54

Summary Calculations

Figure 13: Summary Calculations for the Two-Period Binomial Option Model



Slide 55

Big Time Discussion Here!

Slide 56

Rewriting the Two-Period Option Pricing Model

- We have seen that the call price C_0 can be written as

$$C_0 = \frac{1}{R}[\theta c_{11} + (1-\theta)c_{10}]$$

where $\theta = (R-d)/(u-d)$.

Moreover,

$$c_{11} = \frac{1}{R}[\theta c_{22} + (1-\theta)c_{21}]$$
$$c_{10} = \frac{1}{R}[\theta c_{21} + (1-\theta)c_{20}]$$

- Substituting c_{11} and c_{10} into the expression for C_0, we obtain the two period binomial call option pricing model:

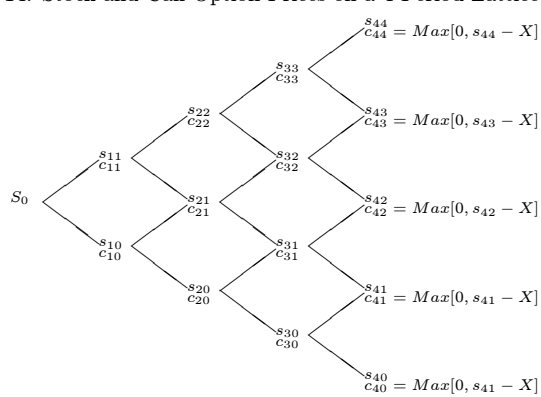
Slide 57

$$C_0 = \frac{1}{R^2}[\theta^2 c_{22} + 2\theta(1-\theta)c_{21} + (1-\theta)^2 c_{20}]$$

Slide 58

The n - Period Binomial Option Pricing Model

Figure 14: Stock and Call Option Prices on a 4-Period Lattice



Slide 59

After n periods the stock price S_n takes on one of the values $s_{n,j}$ for $j = 0, 1, 2, \ldots, n$. The index j refers to the number of upward price movements that have occured over the previous n periods. Since in each period the stock price increases by factor u or decreases by factor d, we have

$$s_{nj} = u^j d^{n-j} S_0$$

The value of the call for each state is known at expiration. That is,

$$\begin{aligned} c_{nj} &= Max(s_{nj} - X, 0) \\ &= Max(u^j d^{n-j} S_0 - X, 0) \end{aligned}$$

Slide 60

- Given the terminal option values, the fair values in periods $(n-1)$ can be computed. Specifically, we have

$$C_{n-1,j} = \frac{1}{R}[\theta c_{n,j+1} + (1-\theta)c_{n,j}] \text{ for } j = 0, 1, 2, \ldots, n-1$$

- This process can be repeated recursively throughout the lattice until the fair value C_0 is obtained.

Slide 61

- As with the two period model, the n period call option price can be written as the present value of the terminal payout assuming a risk neutral economy. In particular, we have:

$$C_0 = \frac{1}{R^n} E_0(C_n)$$

where $R = e^{r\Delta t}$, and the expectation is taken under the assumption that in each period the probability of an upmove is the risk neutral probability θ.

Slide 62

The n period Binomial Option Pricing Equation.

$$C_0 = H^* S_0 - B_0^*$$

where

$$\begin{aligned} H^* &= \frac{1}{R^n} \sum_{j=k}^{n} \binom{n}{j} (u\theta)^j (d(1 -^n \theta))^{n-j} \\ B_0^* &= \frac{X}{R^n} \sum_{j=k}^{n} \binom{n}{j} \theta^j (1-\theta)^{n-j} \end{aligned}$$

and $\theta = (R-d)/(u-d)$

Slide 63

Selecting The Up and Down Parameters

- We need u, d and R
- What happens as $\Delta t \to 0$
- $u = e^{\sigma\sqrt{\Delta t}}$, $d = 1/u$, $R = e^{r\Delta t}$
- we will discuss this later!

Slide 64

Example : Pricing a Call Option Using a 4 - Period Binomial Approximation

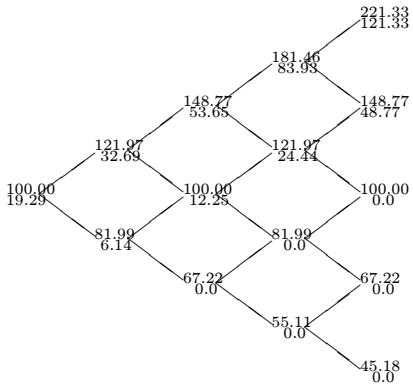
Consider a 1 year at-the-money American call option on a non-dividend paying stock. The stock price is \$100, the risk free rate is 10%, and the annual volatility is 39.72% per year. Using four partitions, $n = 4, T = 1.0$, yields $\Delta t = 0.250$ and

$$\begin{aligned} u &= e^{\sigma\sqrt{\Delta t}} = 1.2197 \\ d &= e^{-\sigma\sqrt{\Delta t}} = 0.8199 \\ \theta &= \frac{e^{r\Delta t} - d}{u - d} = 0.51379 \end{aligned}$$

Slide 65

- At each node the stock price is indicated together with the option price.

Figure 15: Stock and Call Option Prices



Slide 66

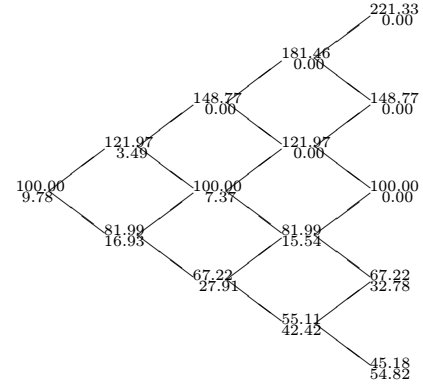
Slide 67

Example: Pricing a Put Option Using a 4-Period Binomial Approximation

- Consider a 1 year at-the-money European put option on the previous stock. Following the backward recursion from the terminal period yields the put prices indicated below the stock price.
- Notice that the option price can fall below the intrinsic value, since early exercise is not permitted.
- Specifically, when the option is deep in-the- money, early exercise may be advantageous since the strike price can be obtained early and can be used to generate interest income.

Slide 68

Figure 16: Stock and Put Option Prices



Slide 69

Pricing American Put Options on a Binomial Lattice

- If the put option in the previous example was American, then clearly its price could never fall below its intrinsic value. In this case the backward recursion algorithm has to be modified.

  Consider vertex (i, j), and assume that all the put prices in period $i+1$ have been obtained. (Initially i is set at the last but one period and the put prices at the terminating dates are given by their intrinsic values.) Then we define the value of the put unexercised as P_{ij}^{GO}, where

$$P_{ij}^{GO} = \frac{1}{R}[\theta P_{i+1,j+1} + (1-\theta)P_{i+1,j}]$$

  The intrinsic value of the option at vertex (i, j) is P_{ij}^{STOP}, where

Slide 70

$$P_{ij}^{STOP} = Max[X - S_{ij}, 0]$$

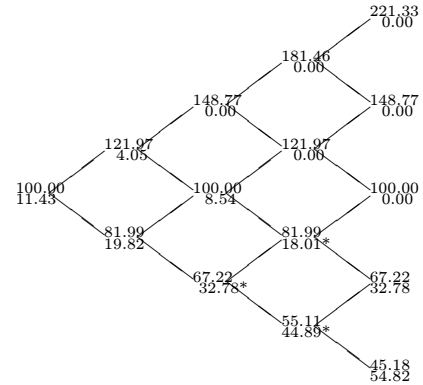
If P_{ij}^{STOP} exceeds P_{ij}^{GO}, then clearly the optimal strategy is to exercise the option rather than continuing to hold it for one more period. Hence the value of the put at vertex (i, j) is just the maximum of these two values. That is

$$P_{ij} = Max[P_{ij}^{GO}, P_{ij}^{STOP}]$$

Slide 71

Example: Pricing an American Put Option

Figure 17: Stock and American Put Option Prices



Slide 72

The Limiting Form of the Binomial Model

As the partition Δt in the above lattices get smaller and smaller, then the price of the European call option converges to the expected terminal value discounted at the riskless rate of return, where the expectation is taken with respect to the risk neutralized process. That is

$$C_0 = \tilde{E}_0 e^{-rT} Max[0, S(T) - X]$$

Slide 73

The Black Scholes Equation

$$C_0 = H^* S_0 - B^*$$

where

$$\begin{aligned} H^* &= N(d_1) \\ B^* &= Xe^{-rT} N(d_2) \end{aligned}$$

and

$$\begin{aligned} d_1 &= [ln(S_0/X) + (r + \sigma^2/2)T]/\sigma\sqrt{T} \\ d_2 &= d_1 - \sigma\sqrt{T}. \end{aligned}$$

$N(x)$ is the probability that a standard normal random variable (usually represented by Z is less than x.

Slide 74

Example: Pricing a European Call Option Using The Black Scholes Model

- Consider the theoretical price of a 3 month call option on a stock priced at \$50. The strike price is 45, the riskless rate is 6% and the volatility is 20% per year. Hence, $S(0) = 50, X = 45, T = 0.25, \sigma = 0.20$, and $r = 6\%$ Then, the Black Scholes call price can be computed as follows.

$$\begin{aligned} d_1 &= [\ln(S_0/X) + (r + \sigma^2/2)T]/\sigma\sqrt{T} = 0.65 \\ d_2 &= d_1 - \sigma\sqrt{T} = 0.426 \end{aligned}$$

Then, using standard normal tables, $N(d_1) = 0.742, N(d_2) = 0.665$, and we obtain

$$\begin{aligned} H^* &= N(d_1) = 0.742 \\ B^* &= Xe^{-rT} N(d_2) = 45e^{(-0.06)(0.25)}(0.665) = 29.48 \\ C(0) &= H^* S(0) - B^* = 50(0.742) - 29.48 = \$7.62 \end{aligned}$$