4.1.2 Using Abstraction to Create an Axiomatic System

The key to unifying di®erent types of items, such as n-vectors and matrices, is to use the following mathematical technique.

> Mathematical Thinking Process
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> Abstraction is the process of taking the focus farther and farther away from speci¯c items by working with general objects. Thus, you become more abstract | hence the term \abstract mathematics."

To illustrate the idea of abstraction in a non-mathematical setting, consider apples and oranges. You can unify these two items into the single comprehensive class of fruits (fruits include apples and oranges as special cases). You can then apply generalization by considering, instead of fruits, the more general class of foods (foods include fruits as a special case). With abstraction, you broaden the class even further by considering objects rather than speci¯c items like foods or fruits. By thinking of objects, you can now include in the same group such diverse items as foods, computers, houses, and much more.

Turning to n-vectors and matrices, you can use abstraction to include these two diverse items in a single group by thinking of objects rather than n-vectors and matrices. That is, you can create a set of objects, say, V. The elements of V can all be n-vectors, matrices, or other items. Thus, the set V contains both n-vectors and matrices as special cases.

Abstraction allows you to unify n-vectors and matrices, but one of the disadvantages of doing so is that you lose the properties of the speci¯c items that give rise to the abstraction. For example, you know how to add two n-vectors u and v, however, you cannot \add" two objects u and v from an arbitrary set V, so a syntax error results when you write

$$u + v$$

You will now see how to overcome this problem.