Mathematical Thinking Process

The progression you have just seen from an ordered list of two numbers to an ordered list of n numbers is an example of a mathematical technique called *generalization*. Generalization is the process of creating, from an original concept (problem, definition, theorem, and so on), a more general concept (problem, definition, theorem, and so on) that includes not only the original one, but many other new ones as well.

Each of the original concepts that gives rise to the generalization is called a *special case*. In the foregoing examples, the ordered lists

$$(73; 175) \quad \text{and} \quad (73; 175; 25)$$

are special cases of an n-vector $u = (u_1; \ldots; u_n)$. The first ordered list, (73, 175), is a special case in which $n = 2$, $u_1 = 73$, and $u_2 = 175$, so $u = (73; 175) \in R^2$. The second ordered list, (73, 175, 25), is a special case in which $n = 3$, $u_1 = 73$, $u_2 = 175$, and $u_3 = 25$, so $u = (73; 175; 25) \in R^3$. Observe that each of the special cases is obtained from the generalization by an appropriate substitution of values. Each special case of an n-vector $u = (u_1; \ldots; u_n)$ is obtained by substituting specific values for n and $u_1; \ldots; u_n$.