Preface to the Instructor

This book is designed as a text for a one-semester introductory undergraduate course in linear algebra. Driven by applications, each chapter begins with a realistic problem to motivate the need for learning the subsequent material. Appropriate theory and technology are then used at the end of the chapter to solve the opening problem. Related project exercises involve the student actively in technology-based problem solving | a feature students have responded to enthusiastically in class testing. Numerous other applications are drawn from physics, statistics, business, and computer science to illustrate where and how the techniques of linear algebra apply.

What makes this book unique, however, is that, in addition to teaching the standard topics of linear algebra, a primary objective is to equip students with general problem-solving skills that are needed in their subsequent mathematics and related courses. This goal is accomplished by identifying and explaining the underlying mathematical *thinking processes* that arise in linear algebra. These thinking processes | hereafter referred to collectively as the *why* mathematics | include uni¯cation, generalization, abstraction, identifying similarities and di®erences, converting visual images to symbolic form and vice versa, understanding and creating de¯nitions and proofs, and developing and working with axiomatic systems. Teaching these ideas explicitly is designed to reduce the time and frustration involved in learning linear algebra and to provide the student with a deeper and more-lasting appreciation of the subject ::: and of mathematics in general.

Teaching Mathematical Thinking Processes

To teach the *why* mathematics, a discussion of such concepts as uni¯cation, generalization, abstraction, and so on, is presented when those ideas arise naturally in the context of linear algebra. Each such concept is named and then referred to whenever the idea appears subsequently in the book.

To understand and to use the techniques of uni¯cation, generalization, abstraction, and developing axiomatic systems, students need to acquire certain basic mathematical skills. These skills include doing proofs, identifying similarities and di®erences, converting visual images to symbolic form (and vice versa), and understanding and creating de¯nitions.

The concept of a proof is fundamental to all advanced mathematics courses. The approach used here, as presented in Appendix A, is adapted from *How to Read and Do Proofs*, second edition, by Daniel Solow and is reprinted with permission from John Wiley & Sons, Inc.

The ability to identify similarities and di®erences among various mathematical concepts is essential to uni¯cation in that like properties from di®erent problems must be isolated, identi¯ed, and then brought together in a single framework. This skill is also used in creating de¯nitions, where it is necessary to identify a common property of all objects being de¯ned.

Much e®ort is also devoted to teaching students how to convert visual images to symbolic form. When students learn mathematics, they typically develop their own ways of imagining and picturing speci¯c concepts | such as the projection of one vector onto another. However, it is one thing to visualize such concepts; it is another thing entirely to translate such an image to symbolic form, especially when quanti¯ers are involved. The art of doing so is illustrated and explained with many examples. Students are also shown how to check for syntax and logic errors that can arise in the translation process. The reverse technique of converting symbolic mathematics to visual form is also taught.

One area in which the student is given special help is in understanding de¯nitions. Although a student may be able to visualize objects with a desirable property | such as linearly independent vectors | it is quite a challenge to create (or even to understand) the symbolic de¯nition. Merely presenting the de¯nition is inadequate because doing so fails to explain how the de¯nition was arrived at and why the de¯nition is correct. The approach taken here is to teach the student to identify similarities shared by all items having the property being de¯ned, and then to translate those observations to symbolic form. They are also taught to verify that the de¯nition includes all objects having the desirable property while excluding all other objects.

Pedagogical Features

The pedagogical features of this book include a realistic introductory problem at the beginning of each chapter to motivate the need for learning the techniques presented in the chapter. Those techniques are then used at the end of the chapter to solve the problem.

Each mathematical thinking process is set o® in the text and is easily identi¯ed by an icon. These thinking processes appear not only in the appropriate chapter summaries but also in their entirety in Appendix B.

Each chapter contains many numbered and titled examples to illustrate the topic under discussion. De¯nitions and theorems have special design features for easy reference. Extensive use of ¯gures provides the student with visual images of mathematical concepts.

Each numbered section is followed by numerous exercises, some of which give the student practice in performing mechanical computations while others test their understanding of the *why* mathematics. The last numbered section in each chapter has exercises that are designed to be solved with MATLAB, Maple, Mathematica, or a graphing calculator. Solutions to exercises whose numbers are in blue are given in the back of the book. The instructor may therefore want to assign for homework those exercises whose numbers are in black.